Important Copyright Notice:

The provision of this paper in an electronic form in this site is only for scholarly study purposes and any other use of this material is prohibited. What appears here is a near-publication draft of the final paper as appeared in the journal or conference proceedings. This is subject to the copyrights of the publishers. Please observe their copyrights.
Distributed Multi-Sensor Fusion Using Generalized Multi-Bernoulli Densities

Wei Yi, Meng Jiang, Reza Hoseinnezhad and Bailu Wang

Abstract

A new method for distributed multi-target tracking with multistatic radar systems is presented. The proposed method is based on using Generalized Covariance Intersection (GCI) of multi-object densities for fusion of the posteriors within a multi-object Bayesian filtering scheme. The presented solution is particularly formulated for sensor fusion with posterior densities that are parametrized as generalized multi-Bernoulli (GMB) distributions which are the unlabeled version of Vo-Vo densities by discarding the labels. To obtain a closed-form solution for fusing GMB densities, we use an efficient approximation to the densities. The approximated density is another GMB density that preserves both the first order moment (intensity or PHD) and the cardinality distribution of the original density. As such, it is called the second-order approximation of the GMB (SO-GMB) density. The resulting explicit expressions for the GCI fusion using SO-GMB approximations allow distributed sensor fusion, not only with Vo-Vo filters but also with Mδ-GLMB and LMB filters being in place as local filters in the multistatic radar system. In two challenging multi-target tracking scenarios, we compare the tracking performance of our proposed method with the recently developed similar technique that was formulated based on fusion of first-order density approximations. The results are followed by a concrete analysis of the advantages of the second-order approximation.

Key words: GCI fusion; RFS; GLMB; GMB

I. INTRODUCTION

Due to their scalability, flexibility, robustness and fault-tolerance, distributed signal processing methods provide unique advantages over similar but centralized techniques. A particular family of such methods are the distributed sensor fusion techniques used for multi-object estimation, or more specifically, multi-target

W. Yi, M. Jiang and B. Wang are with University of Electronic Science and Technology of China, Chengdu, Sichuan, P.R. China, 611731. Fax: +86-028-61830064, Tel: +86-028-61830768, E-mail: kussoyi@gmail.com.
R. Hoseinnezhad is with the School of Aerospace, Mechanical and Manufacturing Engineering, RMIT University, Victoria 3083, Australia, Tel: +61-3-9925-6135, E-mail: rezah@rmit.edu.au.
tracking (MTT). These techniques are at the core of various distributed multi-sensor MTT systems such as multistatic radar systems, mobile sensor networks, vehicular networks and communication networks [1]. Those systems have received significant attention in the last decade, and have been used in a wide range of applications from traffic monitoring to battlefield surveillance.

A distributed sensor fusion solution for multi-target tracking usually includes two major components: (i) an efficient and robust multi-target filter to run locally in each node of the sensor network, independent of the network structure, and (ii) an algorithm for distributed fusion of the information received by each node from multiple other (usually neighboring) nodes, in the absence of any knowledge about the level of correlation between the received information.

In terms of choosing the multi-target filter component of an optimal distributed multi-sensor multi-target tracking system, we note the recent development of the notion of labeled random finite sets and their associated filters [2]. Filters such as Labeled Multi-Bernoulli (LMB) filter [3] and Vo-Vo filter (also called Generalized Labeled Multi-Bernoulli (GLMB) filter)\(^1\) have been of significant interest due to their superior performance in terms of accuracy of cardinality and state estimation of multiple targets with their trajectories. Vo-Vo filter [2], [5] provides a closed-form solution to the optimal Bayesian filter, and has shown to outperform the well-known Probability Hypothesis Density (PHD) filter [6], [7] and its cardinalized version, CPHD filter [8], and the multi-Bernoulli (MB) filter [9] in challenging multi-target tracking scenarios.

This paper focuses on the sensor fusion component of the distributed MTT problem, with particular interest in GLMB family including Vo-Vo, M\(\delta\)-GLMB and LMB filters to be chosen as the local multi-target filter running in each node of the sensor network. An effective information fusion algorithm is expected to combine the information generated by a number of sensor (e.g. radar) nodes and to achieve state estimates and target tracks that are in maximum consistence with all the information obtained from multi-sensor measurements. Because of the unacceptable cost of computing the common information between nodes, optimal fusion [10] is ruled out and one needs to resort to robust suboptimal fusion rules. Mahler [11] proposed the Generalized Covariance Intersection (GCI) fusion rule based on Exponential Mixture Densities (EMDs). Using this rule, both Gaussian and non-Gaussian formed multi-target distributions from different sensors with completely unknown correlation, can be fused sub-optimally.

Following the introduction of GCI fusion rule by Mahler, Clark et al. [12] developed tractable formulations for GCI fusion of multi-target posteriors. This work was followed by particle implementation [13]

\(^1\)We follow the nomenclature used by the pioneer of random set filtering and finite set statistics, Mahler, who used the name “Vo-Vo filter” in his book [4] for the first time.
and Gaussian mixture implementation [14] of distributed fusion of Poisson posteriors (suitable for solution designs involving PHD filters working in each node), then a distributed track-before-detect (TBD) solution based on using local Bernoulli filtering of measurements provided by a Doppler-shift sensor network [15]. Wang et al. [16], [17] recently presented a distributed fusion method with multi-Bernoulli (MB) filters [9], [18]–[20] working in each node, based on GCI rule.

With the recent development of labeled set filters and their advantageous performance compared to previous (unlabeled) random set filters, the design of new distributed sensor fusion systems (with labeled set filters working in each node) is of both fundamental and practical interest. The major task here is to develop tractable algorithms for sufficiently accurate GCI fusion of labeled random set posteriors. This is a challenging task because of the label space mismatching phenomenon [21]; the same realization can be drawn from label spaces of different sensor nodes, which do not have the same implication. To tackle this problem, based on the assumption that all the sensor nodes share the same label space for the birth process, Fantacci et al. [22] have recently proposed a method to implement the GCI fusion with labeled set filters using consistent labels directly. They have presented analytical formulae for distributed fusion using labeled RFSs and the approach performs well in situations where the label spaces of sensor nodes are matching.

Perfect matching of label spaces in different sensor nodes necessitates that the characterizing parameters of all the set label hypotheses in the local Vo-Vo filters are preserved (no pruning). The numerous hypotheses can be then fused using the GCI formulae. For computational tractability, we have to prune the hypotheses with small weights, otherwise the number of hypotheses will grow with time exponentially. Such pruning may then lead to label mismatches. A perfect label space matching after pruning the fused posterior is only possible if all the sensors can observe or detect the same targets (for instance they all have 360° field of view, or they are all directed towards a limited area where targets are maneuvering). In practice, the sensors can have different fields of view, and two sensors may detect two different targets, and pruning of hypotheses including the undetected targets leads to label space mismatches.

An alternative approach is to perform GCI fusion with the unlabeled versions of the distributions in the GLMB family, named as generalized multi-Bernoulli (GMB) family. Wang et al. [17], [21] proposed a tractable solution to the GCI fusion of GMB posteriors via approximating each with a multi-Bernoulli (MB) distribution that matches its first-order moment (its PHD). The approximate MB distribution was also referred to as the first-order approximation of GMB (FO-GMB) distribution.

In this paper, we focus on the above mentioned (second) fusion approach and address the problem of the distributed GCI fusion with labeled set filters. Inspired by the approach through which PHD filter was extended to CPHD [8], and LMB to Mδ-GLMB [23], we present a second order approximation
to a GMB density (SO-GMB) that matches not only its PHD but also its cardinality distribution. Just as the CPHD and $M_\delta$-GLMB filters perform better than PHD and LMB filters (because both preserve the second-order characteristics), we expect that distributed fusion of SO-GMBs performs better than FO-GMBs. We formulate a tractable GCI fusion rule for SO-GMB densities. The fused posterior turns out to be another GMB distribution, and the formula enables sequential fusion within a network of more than two sensor nodes. The proposed SO-GMB fusion method is suitable for labeled set filters including Vo-Vo filter, $M_\delta$-GLMB filter, LMB filter. Gaussian Mixture implementation of the overall distributed fusion algorithm is presented along with a step-by-step pseudocode. We also present how the ranked assignment strategy can be employed to make the implementation tractable for real-time applications.

We have investigated the performance of the proposed distributed sensor fusion method in different multi-target tracking scenarios (varying from simple to very challenging) using multistatic radar systems. Comparative studies with the recent FO-GMB fusion method (which is based on combining first-order approximation of GMB posteriors) were also conducted in each case study. The results indicate that while the performance of SO-GMB fusion is slightly better than FO-GMB fusion in simple tracking scenarios, in challenging situations where the targets are crossing and move in close proximity, SO-GMB fusion performs significantly better.

The outline of the rest of the paper is as follows. Section II provides a brief summary of the labeled random set densities, GLMB and GMB, and GCI fusion scheme. Section III is devoted to the derivation of the second order approximation of a GMB density and presents our distributed GCI fusion using those approximations. Gaussian-mixture implementation of the proposed distributed fusion method and a step-by-step pseudocode of the algorithm are presented in Section IV followed by the comparative simulation results discussed in Section V. Section VI concludes the paper.

II. BACKGROUND

This section reviews the notations commonly used in the random set filtering literature, then provides a brief overview the GLMB and GMB densities as the two recent multi-target densities introduced in the literature. The general GCI fusion rule is then introduced to be revisited later in the next section for derivation of our distributed sensor fusion method.

A. Notation

In this paper, we adhere to the convention that single-target states are denoted by small letters, e.g. $x$, $x$ while multi-target states are denoted by capital letters, e.g. $X$, $X$. Observations generated by single-target states are denoted by the small letter, e.g. $z$, and the multi-target observations are denoted by the capital
letter, e.g. $Z$. Additionally, blackboard bold letters represent spaces, e.g. the state space is represented by $\mathbb{X}$, the label space by $\mathbb{L}$, and the observation space by $\mathbb{Z}$. The collection of all finite sets of $\mathbb{X}$ is denoted by $\mathcal{F}(\mathbb{X})$ and $\mathcal{F}_n(\mathbb{X})$ denotes all finite subsets with $n$ elements.

Symbols for labeled states and their distributions/statistics (single-target or multi-target) are bolded to distinguish them from unlabeled ones, e.g. $x, X, \pi$, etc. To be more specific, the labeled single target state $x$ is constructed by augmenting a state $x \in \mathbb{X}$ with a label $\ell \in \mathbb{L}$.

To evaluate the equality of two sets, vectors or integers, the generalized Kronecker delta function defined as

$$
\delta_Y(X) \triangleq \begin{cases} 
1, & \text{if } X = Y \\
0, & \text{otherwise}
\end{cases}
$$

is used where $X$ and $Y$ can take any of the above mentioned forms. Inclusion relationships of two sets is also evaluated using the following inclusion function:

$$
1_Y(X) \triangleq \begin{cases} 
1, & \text{if } X \subseteq Y \\
0, & \text{otherwise}
\end{cases}
$$

### B. Generalized labeled multi-Bernoulli distribution

The GLMB multi-object distribution was recently formulated and introduced by Vo and Vo [2]. Approximating the prior by this general type of distribution for labeled multiple objects forms the basis of an analytic solution to the Bayes multi-object filter. Under the standard multi-object model, the GLMB is closed under the Chapman-Kolmogorov equation and is also a conjugate prior with the well-known point measurement likelihood function. Thus, with a GLMB prior, the predicted and posterior densities are guaranteed to be GLMB as well.

Let $\mathcal{L} : \mathbb{X} \times \mathbb{L} \to \mathbb{L}$ be the projection $\mathcal{L}((x, \ell)) = \ell$, and $\Delta(X) = \delta_{|X|}(|\mathcal{L}(X)|)$ denote the distinct label indicator. The density of a random finite subset of $\mathbb{X} \times \mathbb{L}$ is GLMB if it can be formulated as

$$
\pi(X) = \Delta(X) \sum_{c \in \mathbb{C}} w^{(c)}(\mathcal{L}(X))[p^{(c)}]^{X}
$$

where $\mathbb{C}$ is a discrete index set. The weights $w^{(c)}(L)$ and the spatial distributions $p^{(c)}(x, \ell)$ satisfy the normalization conditions

$$
\sum_{L \subseteq \mathbb{L}} \sum_{c \in \mathbb{C}} w^{(c)}(L) = 1, \\
\int p^{(c)}(x, \ell) dx = 1.
$$
The GLMB density (3) can be interpreted as a mixture of multi-object exponentials. Note that $\delta$-GLMB, $M\delta$-GLMB and LMB distributions are special cases of GLMB distribution.

C. Generalized multi-Bernoulli distribution

The unlabeled version of GLMB, called Generalized Multi-Bernoulli (GMB) is a multi-object distribution defined in the state space $\mathbb{X}$ and given by [21]:

$$
\pi\left(\{x_1, \ldots, x_n\}\right) = \sum_{\sigma} \sum_{(I, \phi) \in F_n(I) \times \Phi} w^{(I, \phi)} \prod_{i=1}^{n} p^{(\phi),i}(x_{\sigma(i)})
$$

where the summation $\sum_{\sigma}$ is taken over all permutations on the numbers $1, \ldots, n$. The term $\sigma$ denotes any one of the $n!$ possible permutations of the numbers $1, \ldots, n$. Namely, it is a one-to-one correspondence function: $\sigma: \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}$, and $\sigma(i)$ denotes the $i$-th element in a specific permutation. $\Phi$ is a discrete space, $\mathbb{I}$ is the index set of densities and $I^{\mathbb{I}}(i) \in \mathbb{I}^{\mathbb{I}}$ is a vector constructed by sorting the elements of set $I$, $I^{\mathbb{I}}(i)$ denotes the $i$-th element in $I^{\mathbb{I}}$, for example, suppose $I = \{1, 2, 3, 4\}$, then a sorted vector could be $I^{\mathbb{I}} = (1, 2, 3, 4)$ with $I^{\mathbb{I}}(1) = 1, \ldots, I^{\mathbb{I}}(4) = 4$. $w^{(I, \phi)}$ and $p^{(\phi),i}(x)$ satisfy

$$
\sum_{(I, \phi) \in F(I) \times \Phi} w^{(I, \phi)} = 1, \quad (7)
$$

$$
\int p^{(\phi),i}(x) dx = 1, \quad i \in \mathbb{I}. \quad (8)
$$

The distribution of the cardinality of a GMB distributed RFS is given by

$$
\rho(n) = \sum_{(I, \phi) \in F_n(I) \times \Phi} w^{(I, \phi)}. \quad (9)
$$

Accordingly, the PHD becomes

$$
v(x) = \sum_{(I, \phi) \in F(I) \times \Phi} w^{(I, \phi)} \sum_{i \in I} p^{(\phi),i}(x)
= \sum_{i \in \mathbb{I}} \sum_{(I, \phi) \in F(I) \times \Phi} 1_{\mathbb{I}}(i) w^{(I, \phi)} p^{(\phi),i}(x). \quad (10)
$$

Note that GMB distribution is also generalized and can be regarded as GLMB family counterpart that includes the unlabeled version of $\delta$-GLMB, $M\delta$-GLMB and LMB distributions.
D. GCI fusion rule

The GCI fusion rule was proposed by Mahler [11] specifically to enable the fusion of FISST densities in distributed multi-sensor fusion applications. Assume that in a sensor network (e.g. a multistatic radar system), the sensors at two nodes return the point measurement sets \( Z_{1}^{k} \) and \( Z_{2}^{k} \), where \( k \) is the current time. Let us also denote the measurement history at node \( i \) by \( Z_{1}^{k} = (Z_{i}^{1}, \ldots, Z_{i}^{k}) \), and the current multi-object state by \( X^{k} = \{x_{1}^{k}, \ldots, x_{n}^{k}\} \).

At time \( k \), the local multi-object filter at node 1 computes the local multi-object posterior \( \pi_{1}(X^{k} | Z_{1}^{1:k}) \), and receives the locally updated posteriors from a number of other (probably neighboring) nodes in the network. The fusion problem is how to optimally combine the multiple posteriors so that maximum information is preserved in the fused posterior. We are specially interested in a fusion rule that can be implemented in a sequential way. Thus, if one of the nodes communicating its local posterior with node 1 is node 2, the fusion problem is reduced to the problem of combining \( \pi_{1}(X^{k} | Z_{1}^{1:k}) \) and \( \pi_{2}(X^{k} | Z_{2}^{1:k}) \).

According to the GCI fusion rule, a sub-optimal fused distribution is given by the following geometric mean (or exponential mixture) of the local posteriors,

\[
\pi_{\omega}(X^{k} | Z_{1}^{1:k}, Z_{2}^{1:k}) = \frac{\pi_{1}(X^{k} | Z_{1}^{1:k})^{\omega_{1}} \pi_{2}(X^{k} | Z_{2}^{1:k})^{\omega_{2}}}{\int \pi_{1}(X^{k} | Z_{1}^{1:k})^{\omega_{1}} \pi_{2}(X^{k} | Z_{2}^{1:k})^{\omega_{2}} \delta X},
\]

where \( \omega_{1}, \omega_{2} (\omega_{1} + \omega_{2} = 1) \) are the parameters determining the relative fusion weight of each nodes, and the set integral is computed according to [4]

\[
\int f(X) \delta X = \sum_{n=0}^{\infty} \frac{1}{n!} \int f(\{x_{1}, \ldots, x_{n}\})dx_{1} \cdots dx_{n}.
\]

It has been shown that among all exponential mixture densities (EMDs), the density given by equation (11) minimizes the following weighted sum of distances,

\[
\pi_{\omega} = \arg \min_{\pi} (\omega_{1} D(\pi \parallel \pi_{1}) + \omega_{2} D(\pi \parallel \pi_{2})),
\]

where \( D \) denotes Kullback-Leibler divergence (KLD) [14]. For convenience, in what follows we omit the conditioning on the observations and time index \( k \).

III. DISTRIBUTED TRACKING VIA GCI FUSION

Consider a sensor network used in a multi-target tracking application, and a distributed sensor fusion and tracking algorithm in which a Vo-Vo filter (with GLMB assumption for underlying multi-target distributions) or Mδ-GLMB, LMB filter is run locally in each node of the network. To fuse the posteriors coming from neighboring sensor nodes, we assume that local GLMB posteriors are turned into unlabeled
GMB densities before fusion. As it was mentioned earlier, Wang et al. [21] proposed an approximate solution in which each GMB distribution was replaced with an MB density with the same first-order statistical moment. The approximate MB density was called a first order approximation to the GMB density (FO-GMB density) and a closed-form GCI fusion formula was derived to combine two FO-GMB densities. The fused density was shown to turn into a GMB itself.

A. The second-order GMB approximation

Instead of approximating a GMB with an MB density with matching PHD, a closer approximation can be achieved when both the first moment and the entire cardinality distribution are matched. We call such an approximate density as the second-order GMB (SO-GMB) approximate.

**Definition 1.** Consider the GMB density $\pi$ given in (6). A SO-GMB density $\hat{\pi}$ corresponding to $\pi$ is given by:

$$
\hat{\pi}({x_1, \ldots, x_n}) = \sum_\sigma \sum_{I \in F_n(\sigma)} \hat{w}(I) \prod_{i=1}^{n} \hat{p}^{I(\sigma)}(x_{\sigma(i)})
$$

where

$$
\hat{w}(I) = \sum_{\phi \in \Phi} w(I,\phi),
$$

$$
\hat{p}^\sigma(x) = \frac{1}{\hat{w}(I)} \sum_{\phi \in \Phi} w(I,\phi) p^\sigma(\phi,x), \ i \in I.
$$

**Proposition 1.** The SO-GMB density in (14)-(16) preserves both PHD and cardinality distribution of the original GMB density in (6).

**Proof.** The cardinality distribution of the Marginalized GMB density becomes

$$
\hat{\rho}(n) = \frac{1}{n!} \int \hat{\pi}({x_1, \ldots, x_n}) d({x_1, \ldots, x_n})
$$

$$
= \frac{1}{n!} \sum_\sigma \sum_{I \in F_n(\sigma)} \hat{w}(I)
$$

$$
= \sum_{I \in F_n(\sigma)} \hat{w}(I)
$$

$$
= \sum_{(I,\phi) \in F_n(\sigma) \times \Phi} w(I,\phi),
$$

which matches the cardinality distribution $\rho(n)$ given in equation (9).
The PHD is given by

\[ v(x_1) = \sum_{n=0}^{\infty} \frac{1}{n!} \int \pi \left( \{x_1, x_2, \ldots, x_{n+1}\} \right) \, dx_2 \ldots dx_{n+1} \]

\[ = \sum_{n=0}^{\infty} \frac{1}{n!} \int \sum_{\sigma} \sum_{I \in F_{n+1}(\hat{l})} \hat{\lambda}(x) \prod_{i=1}^{n+1} \hat{\rho}^I(x_{\sigma(i)}) \, dx_2 \ldots dx_{n+1}. \]

After substituting the weight terms \( \hat{\omega}(I) \) with their equivalent from equation (15), and the density terms \( \hat{\rho}^I(x_{\sigma(i)}) \) with their equivalents from equation (16), i.e.

\[ \hat{\omega}(I) = \sum_{\phi' \in \Phi} w(I, \phi'), \]

\[ \hat{\rho}^I(x_{\sigma(i)}) = \frac{\sum_{\phi'' \in \Phi} w(I, \phi'') p(\phi'', I^I(x_{\sigma(i)}))}{\sum_{\phi' \in \Phi} w(I, \phi')}, \]

and some algebraic manipulations (reordering the sums and factoring out some terms), the PHD of the SO-GMB turns out to simplify as follows:

\[ v(x_1) = \sum_{I \in F(\hat{l})} \sum_{\phi \in \Phi} w(I, \phi) \sum_{i \in I} p(\phi, i)(x_1), \quad (18) \]

which matches the PHD of original GMB density given by equation (10).

**Remark 1.** Note that the SO-GMB density (14) is also a GMB RFS, especially in the form of the unlabeled version of LMB RFS [21], and provides the prerequisite condition for distributed fusion with GMB distribution. But the number of terms in SO-GMB distribution is substantially lower than GMB distribution. Indeed, the number of components \( (\hat{\omega}(I), \hat{\rho}^I(x)) \) in SO-GMB which need to be stored and computed is \( |F(I)| \) which is substantially smaller than the number of components \( (w(I, \phi), p(\phi, i)(x)) \) in the original GMB given by \( |F(I) \times \Phi| \) for \( w(I, \phi) \) and \( |\Phi| \) for \( p(\phi, i)(x) \).

Before implementing the GCI fusion, one needs to firstly compute the \( \hat{\omega}(I) \) and \( \hat{\rho}^I(x) \) under each hypothesis \( I \) according to (15) and (16), which is similar to the marginalization with respect to the association histories performed in Mδ-GLMB.

**B. GCI fusion of SO-GMB densities**

**Definition 2.** A fusion map (for the current time) is a function \( \tau : \mathbb{I}_1 \rightarrow \mathbb{I}_2 \) such that \( \tau(i) = \tau(i^*) \in \mathbb{I}_2 \) implies \( i = i^* \in \mathbb{I}_1 \). The set of all such fusion maps is called fusion map space denoted by \( \mathcal{T} \). The subset of \( \mathcal{T} \) with domain \( I \) is denoted by \( \mathcal{T}(I) \).
For example, consider two sensors, and their respectively label spaces are $L_1 = \{1, 2\}$ and $L_2 = \{1, 2\}$. According to the Definition 2, there are six fusion maps. Moreover, consider one sensor with label space $L = \{1, 2\}$ and its measurement space $Z = \{z_1, z_2\}$, so there are 13 measurement-track association maps [5]. A comparison of fusion maps and association maps is shown in Fig. 1

Remark 2. The fusion maps play the same role of the measurement-track association map in the Vo-Vo filter [5], with an important difference: they require the cardinalities of the set of tracks in sensor node 2 and the set of tracks in sensor node 1 to be equal.

Proposition 2. The EMD $\pi_\omega(X)$ of the two SO-GMB distributions in (14), can be approximated as a GMB distribution of the form

$$\pi_\omega(\{x_1, \ldots, x_n\}) \approx \sum_{\sigma} \sum_{\mathcal{I} \in \mathcal{F}_n(1_1)} \sum_{\tau \in \mathcal{T}(\mathcal{I})} w_\omega^{(\mathcal{I}, \tau)} \prod_{i=1}^{n} p_\omega^{(\tau, i)}(x_{\sigma(i)})$$

(19)

where

$$w_\omega^{(\mathcal{I}, \tau)} = \sum_{\mathcal{I} \in \mathcal{F}_n(1_1)} \sum_{\tau \in \mathcal{T}(\mathcal{I})} \bar{w}_\omega^{(\mathcal{I}, \tau)}$$

(20)

$$\bar{w}_\omega^{(\mathcal{I}, \tau)} = \hat{w}_1(\mathcal{I})^{\omega_1} \hat{w}_2(\tau(\mathcal{I}))^{\omega_2} \prod_{i \in \mathcal{I}} \int \hat{p}_1^{(\omega_1)}(x)^{\omega_1} \hat{p}_2^{(\omega_2)}(x)^{\omega_2} dx,$$

(21)

$$p_\omega^{(\tau, i)}(x) = \frac{\hat{p}_1^{(\omega_1)}(x)^{\omega_1} \hat{p}_2^{(\omega_2)}(x)^{\omega_2}}{\int \hat{p}_1^{(\omega_1)}(x)^{\omega_1} \hat{p}_2^{(\omega_2)}(x)^{\omega_2} dx}, \quad i \in \mathcal{I},$$

(22)

$$\hat{w}(\mathcal{I}) = \bar{w}(\mathcal{I})^{\omega_1}.$$  

(23)

Proof. From (14), each of the terms $\hat{\pi}_s(X)^{\omega_s}$ in (11), with $s = 1, 2$, can be replaced with

$$\hat{\pi}_s(\{x_1, \ldots, x_n\})^{\omega_s} = \sum_{\sigma} \sum_{\mathcal{I} \in \mathcal{F}_n(1_1)} \hat{w}_s(\mathcal{I}) \prod_{i=1}^{n} \hat{p}_s^{(\tau, i)}(x_{\sigma(i)})^{\omega_s}$$

(24)

where $\hat{w}_s(\mathcal{I}) = \bar{w}_s^{(\mathcal{I})}$. Motivated by [14], [21], [24], we approximate the powered sum with the sum of powers. The validity of this approximation has been well studied in [21]. With this approximation,
Fig. 1: Comparison of fusion maps and association maps with a simple example. (a) fusion maps. (b) association maps.
Finally, by substituting (27) and (28) into (11), we obtain the fused density as given in (19).

\[
\hat{\pi}_s(\{x_1, \ldots, x_n\})^{\omega_s} \approx \sum_{I_1} \sum_{I_2} \cdots \sum_{I_n} \hat{w}_s(I_1)^{\omega_1} \hat{w}_s(I_2)^{\omega_2} \prod_{i=1}^n \left( \frac{\alpha(I_1)}{p_s}\frac{\alpha(I_2)}{p_s} \right)^{\omega_i}.
\]

By substituting (25) into the numerator of (11), we obtain

\[
\hat{\pi}_1(\{x_1, \ldots, x_n\})^{\omega_1} \hat{\pi}_2(\{x_1, \ldots, x_n\})^{\omega_2} = \sum_{I_1} \sum_{I_2} \cdots \sum_{I_n} \hat{w}_1(I_1)^{\omega_1} \hat{w}_2(I_2)^{\omega_2} \prod_{i=1}^n \left( \frac{\alpha(I_1)}{p_1}\frac{\alpha(I_2)}{p_2} \right)^{\omega_i}
\]

Using the notation of a fusion map between two sensor nodes given in Definition 2, (26) can be rewritten as

\[
\hat{\pi}_1(\{x_1, \ldots, x_n\})^{\omega_1} \hat{\pi}_2(\{x_1, \ldots, x_n\})^{\omega_2} = \sum_{\sigma} \sum_{I_1 \in F_1(\sigma)} \sum_{I_2 \in F_2(\sigma)} \hat{w}_1(I_1)^{\omega_1} \hat{w}_2(I_2)^{\omega_2} \prod_{i=1}^n \left( \frac{\alpha(I_1)}{p_1}\frac{\alpha(I_2)}{p_2} \right)^{\omega_i} \int_{I_1} \int_{I_2} \frac{\alpha(I_1)}{p_1} \frac{\alpha(I_2)}{p_2} \alpha(x_1) \alpha(x_2) d\alpha(x_1) d\alpha(x_2)
\]

where the terms \(\hat{w}_1(I_1)^{\omega_1}\) and \(\hat{w}_2(I_2)^{\omega_2}\) have the expressions of (21) and (22). Based on the definition of the set integral defined in (12), the denominator of (11) can be computed as:

\[
\int_{I_1} \int_{I_2} \cdots \int_{I_n} \hat{w}_1(I_1)^{\omega_1} \hat{w}_2(I_2)^{\omega_2} \prod_{i=1}^n \left( \frac{\alpha(I_1)}{p_1}\frac{\alpha(I_2)}{p_2} \right)^{\omega_i} d\alpha(x_1) d\alpha(x_2) \cdots d\alpha(x_n)
\]

Finally, by substituting (27) and (28) into (11), we obtain the fused density as given in (19). \(\blacksquare\)

**Remark 3.** It can be seen from (19) that after GCI fusion of two local posteriors, each hypothesis
Fig. 2: Distributed fusion with SO-GMB filter schematic.

$I_1 \in F_n(I_1)$ generates a set of $|T(I_1)| = \sum_{\tau \in T(I_1)} 1 = \sum_{\sigma_2} \sum_{I_2 \in F_n(I_2)} 1 = n! \times |F_n(I_2)|$ fusion maps. Optimal assignment through all the maps involves computations with complexity of $O(n! \times 2^n)$. In order to reduce the cost of computation, one can perform truncation of fused GMB density using the ranked assignment strategy [5], [25]. The ranked assignment strategy is usually used to solve optimal assignment problems with combinatorial complexities. For example, in the fast implementation of GLMB filter, the ranked assignment strategy was deployed to truncate the multi-target posterior without computing all the hypotheses and their weights [5].

**Remark 4.** The EMD of the two SO-GMB distributions turns out to be another GMB distribution, which can allow the subsequent fusion with another sensor node.

### C. Summary of distributed fusion process

The schematic diagram shown in Fig. 2 shows a graphical representation of the overall distributed fusion process as proposed in this paper. Assuming that at each node of the sensor network, for instance, a Vo-Vo filter is locally run, at each time $k$, a local GLMB posterior is computed in each node using the measurement (e.g. the radar returns) acquired by the sensor in that node. Each node also receives GLMB posteriors from the other nodes which are connected to it in the network. The proposed method
is then applied locally in each node to fuse the local and all received posteriors.

**Remark 5.** The proposed SO-GMB fusion algorithm can accommodate local filters such as Vo-Vo filter, Mδ-GMLB and LMB filter. When local filter is chosen as a Vo-Vo filter and Mδ-GLMB filter, equations (14) and (19) can be employed directly, since their updated distributions are GLMB or Mδ-GLMB densities. If the local filter is an LMB filter, according to the prediction and update equation given in [3], the output of an LMB filter is also an LMB distribution. Note that the proposed SO-GMB fusion is not like LMB filtering which requires the updated distribution to be an LMB RFS, so the SO-GMB fusion can use the pseudo δ-GLMB distribution before the transformation from δ-GLMB density into LMB density that is embedded in LMB update step.

**Remark 6.** Each fused SO-GMB density can also be used to produce trajectories. Once we choose a local sensor node and use its label space as preimage in the fusion map, the fused SO-GMB distribution can inherit and reserve its labels. That is mainly due to the consistency of labels in one sensor node, label space mismatching phenomenon only occurs in different sensor nodes.

IV. GAUSSIAN MIXTURE IMPLEMENTATION

We now detail the Gaussian mixture (GM) implementation of the proposed SO-GMB fusion. Suppose that each single target density $\hat{p}^i(x)$ is a GM in the following form

$$\hat{p}^i(x) = \sum_{i=1}^{J^{(i)}} \omega_i^{(i)} \mathcal{N}(x; m_i^{(i)}, P_i^{(i)}),$$

(29)

where $J^{(i)}$ denotes the number of Gaussian components, $\omega_i^{(i)}$ denotes weights and $\mathcal{N}(x, m, P)$ denotes a Gaussian kernel with its mean located at $m$ and its covariance matrix given by $P$. The product of GMs is a GM and the exponentiation of a GM can be reasonably approximated as a GM [14], then the term $\hat{p}_1^i (x)^{\omega_1} \hat{p}_2^{\tau(i)} (x)^{\omega_2}$ which is an important quantity during the computation of the EMD of two SO-GMBs (see (21) and (22)) is also a GM with the following expression

$$\hat{p}_1^i (x)^{\omega_1} \hat{p}_2^{\tau(i)} (x)^{\omega_2} =$$

\[
\sum_{i=1}^{J^{(i)}} \sum_{j=1}^{J^{(\tau(i))}} \omega_i^{(\omega)} \omega_j^{(\omega)} \mathcal{N}(x; m_{ij}^{(\omega)}, P_{ij}^{(\omega)}),
\]

(30)
where

\[ P_{ij}^{(\omega)} = \left[ \omega_1 \left( P_{ij}^{(1)} \right)^{-1} + \omega_2 \left( P_{ij}^{(2)} \right)^{-1} \right]^{-1} , \]  
\[ m_{ij}^{(\omega)} = P_{ij}^{(\omega)} \omega_1 \left( P_{ij}^{(1)} \right)^{-1} m_{ij}^{(1)} + P_{ij}^{(\omega)} \omega_2 \left( P_{ij}^{(2)} \right)^{-1} m_{ij}^{(2)} , \]  
\[ \omega_{ij}^{(\omega)} = \left( \omega_1^{(i)} \omega_2 \right)^{\omega_1} \left( \omega_2 \omega_2 \right)^{\omega_2} \kappa(\omega_1, P_{ij}^{(1)} \kappa(\omega_2, P_{ij}^{(2)}) \right) \]

\[ \mathcal{N} \left( m_{ij}^{(\omega)} - m_{ij}^{(\omega)} ; 0, \frac{P_{ij}^{(1)}}{\omega_1} + \frac{P_{ij}^{(2)}}{\omega_2} \right) \]

with

\[ \kappa(\omega, P) = \frac{[\text{det}(2\pi P \omega^{-1})]^\frac{1}{2}}{[\text{det}(2\pi P)]^\frac{1}{2}} . \]

By substituting (30) into (22), we obtain

\[ p_{r}^{(1),i}(x) = \frac{\sum_{i=1}^{j(i)} \sum_{j=1}^{j^{(1)}} \omega_{ij}^{(1)} \mathcal{N} \left( x ; m_{ij}^{(1)} , P_{ij}^{(1)} \right)}{\sum_{i=1}^{j(i)} \sum_{j=1}^{j^{(1)}} \omega_{ij}^{(1)}} . \]

Similarly, the term \( \bar{w}_{\omega}^{(I,\tau)} \) in (21) can be rewritten as

\[ \bar{w}_{\omega}^{(I,\tau)} = \bar{w}_1(I)^{\omega_1} \bar{w}_2(\tau(I))^{\omega_2} \prod_{i \in I} \sum_{j=1}^{j^{(1)}} \omega_{ij}^{(1)} \]

and the term \( w_{\omega}^{(I,\tau)} \) in (20) is the normalization of parameter \( \bar{w}_{\omega}^{(I,\tau)} \) over the whole space \{ \( (I, \tau) | I \subseteq I, \tau \in \mathcal{T}(I) \) \}.

Note that each hypothesis \( I \in \mathcal{F}_n(I) \) generates a set of \( |\mathcal{T}(I)| \) fusion maps, which means \( |\mathcal{T}(I)| = |\mathcal{F}_n(I) \times n! | \) fusion maps. In order to reduce the cost of computation, we resort to ranked assignment strategy which has been employed in [5]. The details are as follows.

A. Ranked assignment for fast implementation

Suppose that with \( I_1 = \{ I_1^j(1), \ldots, I_1^j(n) \} \in \mathcal{F}_n(I_1) \) in sensor node 1 and \( I_2 = \{ I_2^j(1), \ldots, I_2^j(n) \} \in \mathcal{F}_n(I_2) \) in sensor node 2, the fusion map \( \tau \) between \( I_1 \) and \( I_2 \) can be represented by an \( n \times n \) assignment matrix \( S \) consisting of 0 or 1 entries with every row and column summing to 1. For \( i, j \in \{1, \ldots, n\} \), \( S_{i,j} = 1 \) if and only if the \( j \)th density \( P_{ij}^{2,j}(x) \) in sensor node 2 is assigned to the \( i \)th density \( P_{ij}^{1,i}(x) \) in sensor node 1, i.e. \( \tau(I_1^j(i)) = I_2^j(j) \).
The cost matrix $C$ of an optimal assignment problem is the $n \times n$ matrix:

$$
C = \begin{bmatrix}
C_{1,1} & \cdots & C_{1,n} \\
\vdots & \ddots & \vdots \\
C_{n,1} & \cdots & C_{n,n}
\end{bmatrix}
$$

(37)

where for $i, j \in \{1, \ldots, n\}$, $C_{i,j} = \mathcal{D}(\hat{p}_1^{(i)}, \hat{p}_2^{(j)})$ is the distance between two pdfs. For Gaussian mixture form (29), we can choose

$$
C_{i,j} = \|H(m_1^{(i)} - m_2^{(j)})\|
$$

(38)

for simplicity, where $m_i^{(i)} = \sum_{i=1}^{J(i)} \omega_i^{(i)} m_i^{(i)}, H$ is the observation matrix and $\| \cdot \|$ denotes norm.

Utilizing ranked assignment strategy, the number of fusion maps $\tau$ between $I_1$ and $I_2$ is reduced from $n!$ to $T$ which is the truncation parameter, i.e. the number of fusion maps with lowest cost in non-decreasing order. $T$ can be chosen as a fixed parameter or a parameter related to corresponding cost.

We can calculate the SO-GMB parameters $\hat{w}^{(I)}$ and $\hat{p}^i(x)$ by utilizing (15) and (16). A pseudo-code of the proposed fusion algorithm is given in Algorithm 1 and the ranked_assignment function means the ranked assignment approach [5], [25].

**Algorithm 1: SO-GMB fusion**

**INPUT:** \{($\hat{w}_1^{(I_1)}, \{\omega_1^{(i)}, m_1^{(i)}, P_1^{(i)}\}$)$_{i=1:J(I_1)}$, ($\hat{w}_2^{(I_2)}, \{\omega_2^{(j)}, m_2^{(j)}, P_2^{(j)}\}$)$_{j=1:J(I_2)}$\}

**OUTPUT:** \{($\hat{w}_\omega^{(I,\tau)}, \{\omega_\omega^{(ij)}, m_\omega^{(ij)}, P_\omega^{(ij)}\}$)$_{i=1:J(I), j=1:J(I)}$\}$_{I \in F(I_1), \tau \in T(I)}$

for $h = 1 : |F(I_1)|$

Find $S = \{I_2 | I_2 \in F(I_2), |I_2| = |I^{(h)}|\}$

for $p = 1 : |S|$ do

Find $C$ using (37) and (38)

{\(\tau(I^{(h)})(p,q)\)}$_{q=1}^T := \text{ranked_assignment}(C, T)$

for $q = 1 : T$ do

for $k = 1 : |I^{(h)}|$ do

For $k$-th single target pdf in the current fusion map $\tau(I^{(h)})(p,q)$, computing all fused GMs:

Find $P_\omega^{(ij)}$ using (31)

Find $m_\omega^{(ij)}$ using (32)

Find $\omega_\omega^{(ij)}$ using (33) and (35)

end

Find $\hat{w}_\omega^{(I,\tau)}$ using (36)

end

end

Find $w_\omega^{(I,\tau)}$ by normalizing weights $\bar{w}_\omega^{(I,\tau)}$ using (20)
V. SIMULATION RESULTS AND DISCUSSION

In two challenging scenarios, we examined the performance of the proposed GCI fusion using SO-GMB distributions, and compared the tracking outcomes (in terms of the resulting OSPA errors [26]) with the recent GCI fusion rule using FO-GMB approximations. With regard to the local filters of the two methods, both the Vo-Vo filters and LMB filters were considered. The fusion weight of each node $\omega_1, \omega_2$ in (11) are both chosen as 0.5. All performance metrics are gathered by averaging over 100 Monte Carlo realizations.

In both scenarios, the single target state includes planar position and velocity,

$$x_k = [p_{x,k} \ p_{y,k} \ \dot{p}_{x,k} \ \dot{p}_{y,k}]^\top$$

and each non-clutter point measurement is a noisy version of the planar position of a single-target,

$$z_k = [z_{x,k} \ z_{y,k}]^\top.$$ 

Thus, the single-target measurement model is given by

$$g_k(z_k|x_k) = \mathcal{N}(z_k; H_k x_k, R_k)$$

$$H_k = \begin{bmatrix} I_2 & 0_2 \end{bmatrix}$$

$$R_k = \sigma_e^2 I_2$$

where $I_n$ and $0_n$ denote the $n \times n$ identity and zero matrices, respectively, $\Delta = 1$ s is the sampling period and $\sigma_e$ is the measurement noise power.

To model single-target motions, the nearly constant velocity model with the following state transition density is used,

$$f_{k|k-1}(x_k|x_{k-1}) = \mathcal{N}(x_k; F_k x_{k-1}, Q_k)$$

$$F_k = \begin{bmatrix} I_2 & \Delta I_2 \\ 0_2 & I_2 \end{bmatrix}$$

$$Q_k = \sigma_v^2 \begin{bmatrix} \Delta^4 I_2 & \Delta^3 I_2 \\ \Delta^3 I_2 & \Delta^2 I_2 \end{bmatrix}$$

where $\sigma_v^2$ is the process noise power. In our simulations, the chosen values are $\sigma_v = 5$ m/s$^2$ and $\sigma_e = 14$ m. The survival probability is $P_{S,k} = 0.98$; target detection in each sensor is independent of the others, and probability of detection at all sensors is $P_D = 0.9$. The number of clutter reports in each scan
is Poisson distributed with $\lambda_c = 15$. Each clutter report is sampled uniformly over the whole surveillance region.

A. Scenario 1

A set of targets move in the two dimensional region $[0, 10000 \text{ m}] \times [0, 10000 \text{ m}]$, all traveling in straight paths with different but constant velocities. The number of targets is time varying due to births and deaths. Specifically, two different targets are born at time $k = 6$ and $k = 11$ respectively, while a target disappears around time $k = 15$. Trajectories of two targets intersect at location $(4000 \text{ m}, 2200 \text{ m})$ at the time $k = 5$, and three targets meet at location $(6000 \text{ m}, 4900 \text{ m})$ at the time $k = 20$. The region and tracks are shown in Fig. 3.

The birth process is LMB with four components [3], all sharing the same probability of existence of $r_B^{(i)} = 0.06$, but having four different Gaussian densities. The Gaussians have the same covariance matrix.
Cardinality Statistics

True
sensor1
sensor2
SO−GMB fusion
FO−GMB fusion

Fig. 4: Cardinality statistics returned by the local sensor nodes, FO-GMB fusion, and SO-GMB fusion in scenario 1. (a) Vo-Vo filters. (b) LMB filters.

Comparative results in terms of cardinality statistics and OSPA error (with parameters \( c = 200 \) m, \( p = 2 \)) are shown in Figs. 4 and 5 with Vo-Vo filters and LMB filters. In each plot, the results returned by each of the two local filters are presented along with those resulted from GCI-fusion of FO-GMB densities and from our proposed GCI fusion of SO-GMB densities.

With the Vo-Vo filters, from Fig. 4 (a), we observe that, all the filters (both local and fusion-based ones) perform similarly in terms of cardinality estimation errors. Fig. 4 (b) shows the cardinality estimation errors based on LMB filters.

Fig. 5 shows that the advantage of sensor fusion is more evident in OSPA errors which are substantially smaller with GCI fusion methods. However, in this scenario, no substantial improvement seems to be obtained with GCI fusion of SO-GMB densities (proposed in this paper) compared to fusion of FO-GMBs. Indeed, a closer look at Fig. 5 (a) would reveal that OSPA errors returned by fusion of SO-GMBs are visibly lower than the ones returned by FO-GMBs at the times of death or when the targets intersect (are located in close proximity of each other).
Fig. 5: OSPA errors returned by the local sensor nodes, FO-GMB fusion, and SO-GMB fusion in scenario 1. (a) Vo-Vo filters. (b) LMB filters.

Fig. 6: Target trajectories in scenario 2. The start/end point for each trajectory is denoted, respectively, by ◯ □.

B. Scenario 2

In order to signify the advantages gained by SO-GMB fusion, triggered by the observation exclaimed in the previous paragraph, scenario 2 was designed to include tracks that are in close proximity most of the times, and include deaths and births. As is shown in Fig. 6, targets 1 and 2 move in almost parallel paths from west to east until time $k=15$ when target 2 disappears. Target 3 is born at time $k=6$ near target 2. The closest distance between targets is 250 m.

With the Vo-Vo filters, the true and estimated cardinalities by both the FO-GMB fusion and SO-GMB
fusion methods, along with the standard deviation of the estimates over 100 Monte Carlo runs, are together presented in Figs. 7 (a) and (b). Comparing the standard deviations of cardinality estimates in the two figures would lead to observation that our proposed distributed sensor fusion method outperforms the recent FO-GMB fusion method in terms of having a smaller standard deviation. With the LMB filters, Figs. 7 (c) and (d) give the similar plots about the true and estimated cardinalities, along with the standard deviation of the estimates. Since the LMB filtering is lacking in cardinality information, both the two fusion methods show larger standard deviations of cardinality estimates than Vo-Vo filters, noting that the SO-GMB fusion method achieves better performance than FO-GMB fusion method especially after the death of target.

Fig. ?? show the average OSPA errors returned by the two fusion methods based on Vo-Vo filters and LMB filters. In agreement with the cardinality errors, substantial reduction in estimation error is observed with SO-GMB fusion compared to FO-GMB fusion.

Overall, the fusion method proposed in this paper appears to perform with better stability and more accuracy, and is more suited to tackle the problems introduced by the cardinality changes.

VI. CONCLUSION

In this paper, we address the problem of distributed multi-target tracking with labeled set filters in the framework of GCI fusion with consideration of label space mismatching phenomenon. Based on the notation of GMB family, firstly, we propose an efficient approximation to the GMB family which preserves both the PHD and cardinality distribution, referred to as second-order approximation of GMB (SO-GMB) density. Then, we derive the explicit formula for GCI fusion of SO-GMB densities and devise a sequential fusion method for distributed target tracking in sensor networks such as bistatic radar
networks. Finally, we compare the recently developed method of GCI fusion of first-order approximation GMB densities with our proposed method in two scenarios. The results show that while both methods perform similarly in simple tracking situations with no frequent targets deaths and intersections between target trajectories, the SO-GMB fusion method is advantageous in more challenging applications where targets move in close proximity with frequent deaths.

ACKNOWLEDGMENT

This work was supported by the Australian Research Council’s Discovery Project Program, via the ARC Discovery Project grants DP130104404 and DP160104662, and supported by the National Natural Science Foundation of China under Grants 61301266, the Chinese Postdoctoral Science Foundation under Grant 2014M550465.

REFERENCES


