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Electric Vehicle Side-Slip Control via Electronic Differential

Abstract: An electronic differential for high-performance electric vehicles with independent driving motors is proposed in this paper. This electronic differential endows the electric vehicle with a close-to-zero vehicle side-slip angle. When vehicle side-slip vanishes, the heading direction of the vehicle coincides with the velocity direction of the mass centre. In addition to the side-slip angle, the yaw rate is driven towards an optimal value with the proposed electronic differential on-board. The improvements in vehicle side-slip and yaw rate responses are of great significance to the handling performance of high-performance vehicles. In this paper, the mathematical relationships between the vehicle dynamic states and the independent motor torques are revealed, based on which the proposed electronic differential controller is designed. Simulation results manifest that in various challenging steering scenarios, the proposed control method outperforms two common electronic differential control schemes in terms of vehicle side-slip and yaw rate responses.

Keywords: electric vehicle; electronic differential; side-slip control; handling dynamics; independent driving motors.

1 Introduction

Most commercialised electronic stability control systems designed for vehicles are braking-based systems, such as Anti-lock Braking System (ABS) and Electronic Stability Program (ESP). The braking-based vehicle stability control systems monitor and control the braking applications exerted on individual wheels. Specifically, ABS is developed to keep the stability-related quantity, wheel slip, in a “safe region” by controlling the braking torque applied on each wheel (Anwar 2006, Hoseinnezhad & Bab-Hadiashar 2011, Shi et al. 2010, Zhang et al. 2010). On the other hand, ESP is designed to help vehicles track the nominal yaw rate as well as the nominal vehicle side-slip angle by applying differential braking torques to the left and right wheels, in such a way that the vehicle is prevented from spinning and drifting out (Kim & Kim 2006, Pi et al. 2011, Rajamani 2012).

The braking-based vehicle stability control systems have been generally successful in maintaining vehicle stability and saving lives in dangerous situations. However, two inherent drawbacks are observed. The additional braking forces generated by such control systems inevitably decrease the vehicle longitudinal velocity/acceleration (Ghike et al. 2009, Osborn & Shim 2004, Sawase & Sano 1999). Besides, these systems are commonly designed to operate only intermittently when the associated vehicle states exceed the thresholds in critically dangerous situations. Therefore, the vehicle states would not be continuously maintained at their desired values.

Recently active vehicle stability control systems, especially direct yaw moment control systems, have attracted increasing attention after the advent of electric vehicles with independent driving motors. Unlike the braking-based stability control systems that apply braking torques to individual wheels, the direct yaw moment control systems independently

control the driving torque distributed to each driving wheel. These control systems enable electric vehicles (with independent driving motors) to continuously track the desired vehicle states by adjusting the independent driving torques (Karogal & Ayalew 2009, Osborn & Shim 2004). When an electric vehicle possesses two independent driving motors, we define the two driving motors along with their control system as an electronic differential system. We use the term “differential”, because the system works in a similar way (but with better performance) to its mechanical counterparts. Apparently an electronic differential system is a type of active vehicle stability control systems.

The vehicle side-slip angle, β , is the angle between the vehicle heading direction (the positive direction of the x -axis in the vehicle local coordinate) and the velocity vector v of the mass centre C, as shown in Figure 1. Two important reasons necessitate the minimisation of vehicle side-slip angle. Firstly, as side-slip angle increases, the yaw moment generated by lateral tyre forces generally descends (Shibahata et al. 1993). At large vehicle side-slip, the generated yaw moment becomes considerably smaller and it can hardly be increased by changing the steer angle. Thus, vehicle tends to lose its stability. Secondly, side-slip angle is normally non-zero during cornering, and drivers naturally assume that the vehicle heading direction is the direction where the vehicle is going. This wrong assumption can mislead the driver into performing excessive or insufficient steering actions. A small vehicle side-slip angle implies consistency of the vehicle heading direction with the velocity vector v , which provides the driver with superior sense of control during cornering (Fu, Hoseinnezhad, Jazar, Bab-Hadiashar & Watkins 2012).

Yaw rate and vehicle side-slip are normally considered as the most important vehicle states that influence vehicle stability and handling (Buckholtz 2002, Chung & Yi 2006, Pi et al. 2011, van Zanten 2000). Recently, an electronic differential control method was proposed in Fu, Hoseinnezhad, Watkins & Jazar (2012), Fu et al. (2014) to control the yaw rate of high-performance electric vehicles. Following on from the results in these two works, we propose in this paper a new control method that minimises vehicle side-slip angle while driving yaw rate close to the desired value associated with neutral steer (this value is explained in Fu, Hoseinnezhad, Watkins & Jazar (2012), Fu et al. (2014)). The merits of having zero vehicle side-slip and close-to-neutral steer characteristic at medium and high speeds are especially of great significance to high performance electric vehicles, such as electric race cars to which vehicle handling is as vital as stability. It should be pointed out that the proposed method is mainly designed for medium and high speed manoeuvres. Firstly, it is at high speeds when the vehicle is more likely to lose stability. At low speeds, vehicles are less likely to lose control and manoeuvrability becomes a major concern instead. Secondly, at low speeds, a large vehicle side-slip angle is necessary to satisfy the tuning kinematics. Thus, when manoeuvring at low speeds, the proposed electronic differential system can be shut down and it reverts to a conventional open differential.

Two very common electronic differential methods that have been introduced in the literature are the equal torque method (Magallán et al. 2008, 2009, 2011) and the Ackerman method (Haddoun et al. 2008, Lee et al. 2000, Perez-Pinal et al. 2009, Zhao et al. 2009). The equal torque method is the most straightforward solution to electronic differential control. It basically emulates the behaviour of an open differential (the most common mechanical differential) by sending identical torque commands to both driving motors. On the other hand, the Ackerman method employs the well-known Ackerman steering geometry, as schematically shown in Figure 2. With the slip-free turning assumption, the Ackerman steering geometry produces the following desired angular velocities for the two driving

wheels:

$$\omega_L = \frac{v_L}{R} = \frac{v_r}{R} \left(1 - \frac{d_r \tan \delta}{2l}\right) \quad (1a)$$

$$\omega_R = \frac{v_R}{R} = \frac{v_r}{R} \left(1 + \frac{d_r \tan \delta}{2l}\right), \quad (1b)$$

where v_r denotes the velocity of the rear axle centre, R represents the tyre radius, d_r denotes the rear track width, l is the wheel base and δ is the steer angle of the front wheels.

In a number of simulations, we compare the proposed method with the equal torque method and the Ackerman method, on a simulated high-performance electric vehicle. Our simulation results demonstrate that the proposed method endows the vehicle with a close-to-zero side-slip angle in challenging steering scenarios whereas the competing schemes fail. Also, with the proposed scheme on-board the yaw rate of the simulated vehicle is driven close to the optimal neutral steer value, while the competing methods still produce sluggish understeer characteristic. Besides, the proposed method maintains the slip ratio (absolute value) of the inner-driving wheel at low values, which does not jeopardise vehicle safety or cause any excessive tyre wear.

The contributions of this paper are twofold. Firstly, we mathematically reveal how the steady-state vehicle side-slip angle is related to the driving motor torque difference, and what the ideal torque difference should be to achieve zero side-slip. We further demonstrate the coupled correlation between the steady-state side-slip angle and the steady-state yaw rate. Secondly, the proposed controller is designed based on a vehicle roll model, which helps attain better robustness. At present, most control methods neglect vehicle roll motion and this overlook may cause the controller to fail at high speed (Smith & Starkey 1994, 1995).

The rest of this paper is organised as follows. In section 2, the vehicle equations of motion are reviewed. In section 3, the relationship between the steady-state vehicle side-slip angle and the driving motor torque difference is presented, followed by the side-slip controller design for the proposed electronic differential. The comparative simulation results are presented in section 4, and the paper concludes in section 5.

2 Vehicle dynamics: Equations of motion

Consider a rear-wheel-drive electric vehicle with a local coordinate frame attached to its mass centre (point C), as shown in Figure 1. The x -axis goes forward horizontally, and the y -axis goes laterally to the left from the driver's view. The z -axis goes upward and makes a right-hand coordinate system. Note that we consider this vehicle configuration since most high-performance vehicles are rear-wheel-drive, but repeating the following derivations for a front-wheel-drive vehicle is straightforward.

The vehicle equations of motion in this coordinate system are as follows:

$$\sum F_x = m\dot{v}_x - mrv_y \quad (2a)$$

$$\sum F_y = m\dot{v}_y + mrv_x \quad (2b)$$

$$\sum M_x = I_x \dot{p} \quad (2c)$$

$$\sum M_z = I_z \dot{r}, \quad (2d)$$

where m is the vehicle mass, v_x and v_y denote the longitudinal and lateral velocities of the mass centre respectively, p and r denote the roll and yaw rates respectively, I_x and I_z are the roll and yaw moments of inertia respectively. As mentioned previously, we employ a vehicle roll dynamic model (instead of a planar one) to take into account the vehicle roll motion, in order to achieve more realistic simulation results and better control performance.

Expanding the left-hand side of the equations in (2), we obtain the following force system exerted on the electric vehicle:

$$\sum F_x = \sum_{i=1}^4 (F_{xi} \cos \delta_i - F_{yi} \sin \delta_i) \quad (3a)$$

$$\sum F_y = \sum_{i=1}^4 (F_{xi} \sin \delta_i + F_{yi} \cos \delta_i) \quad (3b)$$

$$\sum M_x = - \sum_{i=1}^4 z_i (F_{xi} \sin \delta_i + F_{yi} \cos \delta_i) + M_k + M_c \quad (3c)$$

$$\sum M_z = \sum_{i=1}^4 x_i (F_{xi} \sin \delta_i + F_{yi} \cos \delta_i) - \sum_{i=1}^4 y_i (F_{xi} \cos \delta_i - F_{yi} \sin \delta_i), \quad (3d)$$

where x_i , y_i and z_i represent the coordinates of the i th wheel, δ_i stands for the steer angle of the i th wheel, M_k and M_c denote the roll moments produced by the springs and dampers of the suspension system respectively, F_{xi} and F_{yi} are the longitudinal and lateral tyre forces exerted on the i th wheel respectively. Note that $\delta_3 = \delta_4 = 0$, and δ ($\cot \delta = (\cot \delta_1 + \cot \delta_2)/2$) is used in place of δ_1 and δ_2 for simplicity.

The roll moments M_k and M_c can be further expressed by the equations below:

$$M_k = -k\phi \quad (4a)$$

$$M_c = -c\dot{\phi} = -cp, \quad (4b)$$

where the coefficients k and c are the total roll stiffness and total roll damping of the vehicle suspension system respectively, and ϕ denotes the roll angle of the vehicle.

It is important to note that for the sake of simplicity, in the above dynamic equations, the wheel lateral inertia is neglected. Consequently, the small differences between the forces acting on the body and the tire-ground forces are neglected and the roll centres of front and rear suspensions are assumed to be at the same height as wheel centers. The detailed explanations of the above vehicle equations of motion (equations (2a)–(4b)) are available in Jazar (2014).

The longitudinal and lateral tyre forces, F_{xi} and F_{yi} , are non-linearly dependent on the wheel slip ratio s_i , tyre side-slip angle α_i and tyre normal load N_i . These forces are elaborated by the well-known *Pacejka Magic Formula* equations (Pacejka 2012a) which have been widely used in vehicle dynamic analysis such as Hu et al. (2012), Mutoh et al. (2008) and Mutoh & Nakano (2012):

$$y(x) = D \sin[C \arctan\{Bx - E(Bx - \arctan Bx)\}] \quad (5)$$

with

$$Y(X) = y(x) + S_V \quad (6a)$$

$$x = X + S_H \quad (6b)$$

where X represents the input variable $\tan \alpha_i$ or s_i , $Y(X)$ denotes the output variable F_{xi} or F_{yi} , S_H and S_V are the horizontal shift and vertical shift, respectively, B , C , D and E are the stiffness factor, shape factor, peak value and curvature factor, respectively. Note that the equations and notations of the *Pacejka Magic Formula* are directly copied from Pacejka (2012a) and should not be mistaken for similar notations used for other variables in this paper.

The detailed expressions for B , C , D , E , S_H and S_V are available in Pacejka (2012a). The calculation of these parameters requires the value of the wheel slip ratio s_i , tyre side-slip angle α_i and tyre normal load N_i . The wheel slip ratio s_i used in the *Magic Formula* is defined as (Pacejka 2012b):

$$s_i = \frac{R\omega}{v_{xi}} - 1. \quad (7)$$

where R represents the tyre radius, ω stands for the wheel angular velocity, and v_{xi} denotes the velocity of the i th wheel centre in the wheel heading direction which is computed as follows:

$$v_{xi} = (v_x - y_i r) \cos \delta_i + (v_y + x_i r) \sin \delta_i. \quad (8)$$

The tyre side-slip angle α_i of each tyre is expressed as:

$$\alpha_i = \arctan \frac{v_y + x_i r}{v_x - y_i r} - \delta_i - C_{\delta_i} \phi, \quad (9)$$

where C_{δ_i} is called the roll steer coefficient. The tyre normal load N_i , considering longitudinal and lateral load transfers, can be calculated by the following equations:

$$N_1 = \frac{m}{2l}(gl_r - ha_x) - \frac{1}{d_f}(k_{\phi f}\phi + c_{\phi f}p + m_s a_y e_f \frac{l_r}{l} + m_{uf} a_{yf} h_f) \quad (10a)$$

$$N_2 = \frac{m}{2l}(gl_r - ha_x) + \frac{1}{d_f}(k_{\phi f}\phi + c_{\phi f}p + m_s a_y e_f \frac{l_r}{l} + m_{uf} a_{yf} h_f) \quad (10b)$$

$$N_3 = \frac{m}{2l}(gl_f + ha_x) + \frac{1}{d_r}(k_{\phi r}\phi + c_{\phi r}p + m_s a_y e_r \frac{l_f}{l} + m_{ur} a_{yr} h_r) \quad (10c)$$

$$N_4 = \frac{m}{2l}(gl_f + ha_x) - \frac{1}{d_r}(k_{\phi r}\phi + c_{\phi r}p + m_s a_y e_r \frac{l_f}{l} + m_{ur} a_{yr} h_r), \quad (10d)$$

where l represents the wheel base, l_f and l_r denote the distances from the front axle and rear axle to the vehicle mass centre respectively, h denotes the vehicle mass centre height, h_f and h_r are the front and rear unsprung mass centre heights respectively, d_f and d_r stand for the front and rear track widths respectively, $k_{\phi f}$ and $k_{\phi r}$ are the front and rear suspension roll stiffnesses respectively, $c_{\phi f}$ and $c_{\phi r}$ are the front and rear suspension roll dampings respectively, e_f and e_r are the front and rear roll centre heights respectively, a_x and a_y represent the longitudinal and lateral accelerations at the vehicle mass centre respectively, a_{yf} and a_{yr} stand for the lateral accelerations at the front and rear unsprung mass centres respectively, m_s , m_{uf} and m_{ur} denote the sprung mass, front unsprung mass and rear unsprung mass respectively.

Equations (2)-(10) constitute a complete non-linear vehicle dynamic model which thoroughly describes the vehicle motions. As it will be presented in section 4, this complete model has been fully implemented in MATLAB/Simulink environment for the simulation studies in this paper.

3 Side-slip control via electronic differential

3.1 Side-slip response formulation

When the tyre side-slip angle α_i is small, the *Pacejka Magic Formula* equation for lateral tyre force can be considered to be linearly proportional to α_i , which reads:

$$F_{yi} = -C_{\alpha i} \alpha_i \quad (11)$$

where $C_{\alpha i}$ is the tyre cornering stiffness. Note that the vehicle body roll motion causes the wheel camber angle to change, which in turn results in an additional tyre camber thrust. To accommodate the effect of camber angle change, the total lateral tyre force is modified as follows:

$$F_{yi} = -C_{\alpha i} \alpha_i - C_{\phi i} \phi \quad (12)$$

where $C_{\phi i}$ represents the tyre camber thrust coefficient (Jazar 2014). When β , δ_i and α_i are small, the expression of α_i (equation (9)) simplifies to:

$$\alpha_i = \beta + x_i \frac{r}{v_x} - \delta_i - C_{\delta i} \phi. \quad (13)$$

We assume that the left and right longitudinal tyre forces are initially symmetric, namely $F_{x1} = F_{x2}$ and $F_{x3} = F_{x4}$, and that δ_i is small (thus $\sin \delta_i \approx 0$ and $\cos \delta_i \approx 1$). Combining equations (2)-(13), we obtain a linearised version of the equations of motion that govern the lateral, yaw and roll motions of the car:

$$C_r r + C_p p + C_\beta \beta + C_\phi \phi + C_\delta \delta = m \dot{v}_y + m r v_x \quad (14a)$$

$$D_r r + D_p p + D_\beta \beta + D_\phi \phi + D_\delta \delta = I_z \dot{r} \quad (14b)$$

$$E_r r + E_p p + E_\beta \beta + E_\phi \phi + E_\delta \delta = I_x \dot{\phi}. \quad (14c)$$

The vehicle longitudinal motion is neglected in (14), since in vehicle handling analysis the vehicle longitudinal velocity v_x is commonly maintained constant to reveal fundamental lateral and yaw behaviours. All coefficients on the left-hand side of the equations in (14) are vehicle parameters that can be calculated and are normally assumed to be time-invariant in vehicle dynamic analysis. The detailed explanations for these coefficients are available in Jazar (2014).

So far, we have applied small angle assumption a few times in the above derivation. It is worth noting that even in intense driving conditions, the usage of small angle assumption in linearising the equations of motion is justified. The justification of its application to the tyre side-slip angle α_i can be easily explained by an example given in Milliken & Milliken (1995): a typical racing tyre inflated at 31 psi for a given load of 1800 lb. This Goodyear racing tyre produces the maximum lateral tyre force at a tyre side-slip angle of about 6.5° after which the tyre enters the unstable frictional range. Note that 6.5° is only about 0.1 rad and the tyre mostly operates in the range below 6.5° , which enables us to safely apply small angle assumption in the linearisation. As for the steer angle δ_i , we observe from equation (9) that α_i is directly dependent on the value of δ_i . If δ_i goes too high, the magnitude of α_i is very likely to become large as well, and in turn the vehicle may lose stability. In other

words, in the stable region the steer angle δ_i is normally low. At a medium speed, say 15 m/s (54 km/h), a front wheel steer angle of 0.1 rad (used in the simulation studies in section 4) is a typical magnitude with which the vehicle is still stable. Besides, when both α_i and δ_i are assumed to be small, the associated vehicle side-slip angle β will be small as well. These justifications imply that the mathematical relationships derived from the linearised models still hold in intensive driving conditions and can capture the main characteristics of vehicle dynamics.

It is important to note that the linearised equations are only used to derive the mathematical relationships between the vehicle states and the motor torques. However for simulation studies, the complete set of non-linear equations (2)-(10) are employed to model the electric vehicle.

With the proposed electronic differential on-board, we are able to send different torque commands to the two driving motors, so that the tyre forces F_{x3} and F_{x4} can be different. Denoting this difference by $\Delta F_x = F_{x3} - F_{x4}$, we note that such a difference is equivalent to an additional moment $\Delta M = \Delta F_x \times d_r/2$ applied on the rear axle plus a force ΔF_x exerted at the rear axle centre, as illustrated in Figure 3. Here, d_r denotes the rear track width. Thus, in presence of a difference between the longitudinal tyre forces, only the second equation in (14) needs to be modified as follows:

$$D_r r + D_p p + D_\beta \beta + D_\phi \phi + D_\delta \delta + \frac{d_r}{2} \cdot \Delta F_x = I_z \dot{r}. \quad (15)$$

Note that the lateral, yaw and roll dynamics of the vehicle are substantially slower than the dynamics of the electric motors. Therefore, in the context of generating motor control commands, the time-derivative terms in the equations of motion (14) are negligible. Thus, for controlling the driving motors we can safely use the following steady-state form of the motion equations:

$$\begin{bmatrix} C_\beta & C_r - mv_x & C_\phi \\ D_\beta & D_r & D_\phi \\ E_\beta & E_r & E_\phi \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \phi \end{bmatrix} = \begin{bmatrix} -C_\delta & 0 \\ -D_\delta - \frac{d_r}{2} \\ -E_\delta & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \Delta F_x \end{bmatrix}. \quad (16)$$

Solving the above system of equations yields the following vehicle side-slip (steady-state) response in terms of the control inputs δ and ΔF_x :

$$\beta = \frac{Z_1}{Z_0} \delta + \frac{Z_2}{Z_0} \Delta F_x, \quad (17)$$

where,

$$Z_0 = E_\beta(D_\phi C_r - D_r C_\phi - mv_x D_\phi) + E_\phi(D_r C_\beta - D_\beta C_r + mv_x D_\beta) + E_r(D_\beta C_\phi - D_\phi C_\beta) \quad (18a)$$

$$Z_1 = E_r(D_\phi C_\delta - D_\delta C_\phi) + E_\phi(D_\delta C_r - D_r C_\delta - mv_x D_\delta) + E_\delta(D_r C_\phi - D_\phi C_r + mv_x D_\phi) \quad (18b)$$

$$Z_2 = \frac{d_r}{2} (E_\phi C_r - E_r C_\phi - mv_x E_\phi). \quad (18c)$$

Equation (17) presents a direct relationship between the steady-state vehicle side-slip angle and the rear tyre force difference ΔF_x . In steady-state cornering, the side-slip angle is

normally non-zero. As mentioned previously in section 1, two important reasons necessitate the minimisation of vehicle side-slip. Thus, our approach is based on regulating the steady-state side-slip angle expressed by equation (17) at zero. As a result, the vehicle heading direction becomes consistent with the vehicle velocity direction, and this consistency improves vehicle handling and the driver's sense of control.

The longitudinal tyre forces applied on the tyre contact patches are directly related to the torques produced by the driving motors, according to the following torque equilibrium equation in the driving wheel coordinate:

$$T = J \frac{d\omega}{dt} + F_x R + F_z a, \quad (19)$$

where T represents the motor torque, J denotes the mass moment of inertia of the driving wheel assembly, F_x stands for the longitudinal tyre force exerted on the contact patch, F_z is the normal reaction force applied by the ground, and a is the tyre pneumatic trail.

Again, compared to the dynamics of the electric motors, the dynamics of the mechanical components is much slower. More precisely, during each sampling time of the electronic control system, the variation of the wheel angular velocity ω is so small that ω can be considered constant. Besides, the term $F_z a$ is normally small compared to $F_x R$ and it is also neglected. Thus, we can simplify equation (19) to:

$$T = F_x R. \quad (20)$$

Equation (20) reveals that the longitudinal tyre forces can be directly controlled by tuning the torque commands sent to the driving motors.

From equation (20), one easily derives that:

$$\Delta T = \Delta F_x R. \quad (21)$$

Substituting equation (21) in equation (17) produces:

$$\beta = \frac{Z_1}{Z_0} \delta + \frac{Z_2}{Z_0 R} \Delta T. \quad (22)$$

Let ΔT^* be the desired motor torque difference that achieves $\beta^* = 0$, then we have:

$$\beta^* = \frac{Z_1}{Z_0} \delta + \frac{Z_2}{Z_0 R} \Delta T^* = 0, \quad (23)$$

and

$$\Delta T^* = -\frac{Z_1 R}{Z_2} \delta. \quad (24)$$

Equations (22)-(24) lay the theoretical foundations for the controller design presented in the following subsection.

3.2 Controller design

Subtracting equation (22) from equation (23), we obtain the following relationship expressed in terms of errors:

$$\beta^* - \beta = \frac{Z_2}{Z_0 R} (\Delta T^* - \Delta T). \quad (25)$$

Rearrangement of equation (25) leads to:

$$\Delta T^* - \Delta T = \frac{Z_0 R}{Z_2} (\beta^* - \beta). \quad (26)$$

We note that the electronic differential system is actually a discrete control system. The output of the controller $\Delta T(k+1)$ at discrete time $k+1$ must be generated to make $\beta(k)$ approach β^* ($\beta^* \equiv 0$) as soon as possible. Therefore, our control policy is to create $\Delta T(k+1) = \Delta T^*$, which leads to:

$$\Delta T(k+1) - \Delta T(k) = \frac{Z_0 R}{Z_2} (\beta^* - \beta(k)). \quad (27)$$

Dividing both sides by the sampling time t_s yields:

$$\frac{\Delta T(k+1) - \Delta T(k)}{t_s} = \frac{Z_0 R}{Z_2 t_s} (\beta^* - \beta(k)). \quad (28)$$

Because the sampling time t_s is very small, so the left-hand side of equation (28) can be considered as the time-derivative of the torque difference ΔT . Thus, integrating both sides of equation (28) in continuous time t provides:

$$\Delta T(t) = \frac{Z_0 R}{Z_2 t_s} \int_0^t e_\beta(\tau) d\tau, \quad (29)$$

where,

$$e_\beta(\tau) = \beta^* - \beta(\tau) = -\beta(\tau). \quad (30)$$

Equation (29) indicates that the ideal torque difference between the two driving motors can be achieved by a simple I controller. The control gain of this controller, $\frac{Z_0 R}{Z_2 t_s}$, is indeed speed-dependent as the parameters Z_0 and Z_2 depend on v_x , according to equations (18a) and (18c). At every discrete time k , the speed v_x should be estimated and fed to the controller to compute this control gain. In case of a bounded estimation error Δv_x , this error could cause the control gain in equation (29) to change accordingly and results in an increment in the control command $\Delta T(k)$ at discrete time k . This increment would lead the vehicle side-slip angle $\beta(k)$ to increase as well. Since the proposed scheme is a feedback controller, the increased $\beta(k)$ would decrease the control command $\Delta T(k+1)$ at time $k+1$. As a result, the vehicle side-slip angle $\beta(k+1)$ will decrease and the effect of this estimation error Δv_x can be compensated for.

Note that the Simulink model used for simulation studies employs the complete set of non-linear equations (2)-(10), but we linearised these equations and neglected some insignificant dynamics to achieve the above results. In view of these modelling errors, we propose a PID controller, instead of just an I controller, to better regulate the vehicle side-slip.

3.3 Controller effect on yaw rate

It is important to notice that if we solve equation (16) for yaw rate r , we will obtain a similar expression to equation (17), as follows:

$$r = \frac{Z_3}{Z_0}\delta + \frac{Z_4}{Z_0}\Delta F_x, \quad (31)$$

where,

$$Z_3 = E_\beta(D_\delta C_\phi - D_\phi C_\delta) + E_\phi(D_\beta C_\delta - D_\delta C_\beta) + E_\delta(D_\phi C_\beta - D_\beta C_\phi) \quad (32a)$$

$$Z_4 = \frac{d_r}{2}(E_\beta C_\phi - E_\phi C_\beta). \quad (32b)$$

Substituting equation (21) in equation (31) leads to:

$$r = \frac{Z_3}{Z_0}\delta + \frac{Z_4}{Z_0 R}\Delta T. \quad (33)$$

Equation (33) indicates that when ΔT is tuned to obtain a certain vehicle side-slip angle governed by equation (17), the yaw rate of the vehicle will be influenced as well. Therefore, when designing a controller to regulate the vehicle side-slip, the controller parameters have to be carefully tuned in such a way that not only is a desirable vehicle side-slip performance achieved, but also the yaw rate performance is satisfactory. This parameter tuning often involves appropriate trade-off between the vehicle side-slip and yaw rate responses. In the simulation studies in section 4, we shall show that with the configuration of the simulated vehicle, both the yaw rate and vehicle side-slip can be optimised at the same time. However, it should be pointed out that simultaneous optimisation is not always possible and it depends on the configurations of the particular vehicle. If the requirement of vehicle side-slip minimisation contradicts the need of yaw rate optimisation, appropriate trade-off has to be made and the yaw rate response may not achieve the ideal value, in turn the vehicle presents understeer of some extent.

3.4 Complete control algorithm

Figure 4 shows the schematic of the proposed electronic differential. The electronic differential control system consists of two PID type controller units. The side-slip controller unit generates half the difference in torque commands, $\Delta T/2$, from the side-slip error (the difference between the desired side-slip angle and its actual value), and the speed controller unit provides the base torque T_{base} which is the average of the two torque commands. We tune the base torque in such a way that the vehicle longitudinal velocity, v_x , follows the desired one v_x^* read from the throttle pedal sensor (Karogal & Ayalew 2009). In simulations, the throttle pedal is held at a fixed position to keep v_x constant. The outputs of the two controllers are summed up and subtracted to form the left and right torque commands T_L and T_R . The two inverters receive these commands and convert them to electric signals, in conjunction with the feedback phase signal φ read from the motor encoders, to drive these two brush-less permanent-magnet DC motors.

The actual longitudinal velocity v_x and vehicle side-slip β can hardly be measured physically by any sensors at low cost and they need to be estimated by vehicle state observers.

The longitudinal velocity v_x can be estimated using one of the methods proposed in Imsland et al. (2006). Besides, many vehicle side-slip estimation methods have been proposed in the literature such as Piyabongkarn et al. (2009), Pi et al. (2011), Doumiati et al. (2011), Fukada (1999) which can be readily employed in the proposed control system. In reality, the actual (estimated) and desired longitudinal velocities v_x and v_x^* , as well as the actual (estimated) vehicle side-slip β are fed back to form the errors for the two PID controller units. However in the simulation studies, we employed accurate v_x and β for all the competing control methods, in order to show how the proposed controller works without parameter uncertainties (i.e. estimation errors) and how it outperforms the competing methods in principle.

4 Simulation results

In order to verify the effectiveness of the proposed control scheme, we have conducted a number of simulations in MATLAB/Simulink environment. As mentioned in section 2, a complete non-linear vehicle dynamic model is employed for simulation studies. The vehicle parameters used in our simulations are from a real electric race car designed and built at our institution, as shown in Figure 5. The parameters are listed in Table 1. This car is the third generation all-electric race car developed at our institution, which is featured with two independent driving motors for rear wheels. As our institution is a member of the Formula SAE Tire Test Consortium (TTC), we obtained real tyre testing data from TTC and we employed those data in *Pacejka Magic Formula* for tyre force calculations in our simulations (Kasprzak & Gentz 2006).

In the following simulation analysis, we compare the proposed method with other two popular control methods: the equal torque method and the Ackerman method, as mentioned in section 1. Our simulations comprise two sections: simulations with step steering inputs and simulations with sinusoidal steering inputs. In each section, we examine the vehicle side-slip and yaw rate performances of a fully simulated electric vehicle equipped with any of the above three electronic differential designs. Also, the slip ratio of the inner-driving wheel, which normally presents the worst slip among the four wheels, is assessed in each simulation study.

4.1 Simulations with step inputs

In this section, step inputs are employed as the steering inputs to the simulated vehicle. We have examined a large range of possible step magnitudes. For the sake of brevity, here we only present the simulation results for step inputs $\delta = 0.1$ rad and $\delta = 0.12$ rad. The initial vehicle longitudinal velocity v_x is chosen as 15 m/s (54 km/h) and will be maintained constant by the speed controller unit. With this longitudinal speed, the two selected steering inputs represent rather challenging cornering scenarios. To clearly show the transients, in our simulations the step steering commands occur at $t = 10$ s.

Figure 6 shows the vehicle side-slip angle responses versus time with the three different control methods on-board, when the steering input is $\delta = 0.1$ rad. We observe that all three side-slip angle curves converge to some certain values very quickly after the steering input occurs, but the one representing the proposed method is smaller than the other two. Indeed, using the proposed method, we gain a steady-state side-slip angle of about 0.005 rad, while with the equal torque method and the Ackerman method on-board, the side-slip angles

increase to about 0.0135 rad and 0.016 rad, respectively. The vehicle side-slip β , with the proposed control method on-board, is the smallest one among these three. In other words, the vehicle heading direction is closer to the vehicle velocity direction of the mass centre, and the driver can handle the vehicle more easily with a more accurate sense of steering.

Another benefit that our proposed control method brings to the simulated vehicle is that when the two PID controller units are tuned properly, the yaw rate can be driven close to the desirable value associated with neutral steer, along with the suppression of vehicle side-slip. This means that the normal sluggish understeer characteristic is attenuated and a better cornering agility is achieved. As can be seen in Figure 7, the yaw rate of the vehicle with the proposed electronic differential on-board is almost the same as the ideal value, when $\delta = 0.1$ rad. However, the other two curves are much lower than the desired value, which means that the vehicle presents sluggish understeer characteristic. When designing an electronic differential for a high-performance electric vehicle, this benefit becomes more important because sports cars normally tend to have a neutral steer or even slight oversteer characteristic (Fu, Hoseinnezhad, Watkins & Jazar 2012, Fu et al. 2014) to enhance cornering agility.

Figure 8 shows the slip ratio responses of the inner-driving (inner-rear) wheel with the three methods on-board, when $\delta = 0.1$ rad. The inner-driving wheel normally presents the worst wheel slip because it is considerably unloaded by the centrifugal force during cornering. In Figure 8, we see that the slip ratio of the equal torque method is always positive, while the slip ratios of the other two are negative. Besides, the slip ratio of the proposed method is larger than the competing methods in absolute value. These results indicate that the longitudinal tyre force applied on the inner-driving wheel is positive (forward) and small for the equal torque method, and the tyre force generated by the Ackerman method is negative (backward) and small. However, using the proposed method, a large (in absolute value) negative (backward) longitudinal tyre force is generated to decrease vehicle side-slip angle and increase yaw rate. This is verified by Figure 9 in which the longitudinal forces exerted on the inner-driving wheel are clearly portrayed. It is crucial to notice that the slip ratio with the proposed controller on-board is still in a very safe range, even though its absolute value is larger than the other two. Neither does this slip ratio jeopardise vehicle safety nor causes any excessive tyre wear.

With brush-less permanent-magnet DC motors, both positive and negative torques can be generated. When motors generate negative torques, they are actually working in the electrical braking mode (Mutoh 2012). We observe from Figure 9 that the longitudinal tyre force on the inner-driving wheel with the proposed controller on-board is negative, while the wheel angular velocity of the inner-driving wheel seen from Figure 10 is positive. This means that the direction of the motor torque is opposite to the direction of the angular velocity of the inner-driving wheel, namely this driving motor is operating in the electrical braking mode. Since the vehicle longitudinal velocity v_x is maintained by the speed controller unit that generates the base torque T_{base} , the longitudinal velocity v_x will not be compromised due to this electrical braking motion. Moreover, in the electrical braking mode, regenerative braking can be made possible to enhance the efficiency of the driving system.

Figure 11 plots the vehicle side-slip responses using different control schemes when the step steering input is pushed up to a more challenging case, $\delta = 0.12$ rad. As shown in this figure, the proposed method outperforms the other two control methods in terms of leading to the smallest side-slip angle. In steady state, the equal torque method makes the vehicle side-slip angle almost five times larger than the one produced by the proposed

method. The Ackerman method makes the vehicle side-slip diverge rapidly, and the vehicle loses its stability very quickly.

Accordingly, the yaw rate response curves plotted in Figure 12 demonstrate similar trends. The yaw rate of the electric vehicle with the proposed controller on-board converges to a certain value after a short period of oscillation, and it is the closest one to the desired value among the three curves. The curve of the equal torque method converges as well but it is further away from the ideal curve. Again, the yaw rate produced by the Ackerman method diverges, which is consistent with the vehicle side-slip response shown in Figure 11.

Figure 13 shows the slip ratio responses of the inner-driving wheel. Similarly, the slip ratio diverges very quickly when we apply the Ackerman method, while the other two methods quickly stabilise the slip ratio. Although oscillation appears at the beginning of the cornering with the proposed controller on-board, the peak values of this oscillation (absolute value) are very small. Thus, this oscillation does not cause any instability and excessive tyre wear.

In Table 2, we present the average errors of the vehicle side-slip angle and yaw rate for $\delta = 0.1$ rad. The average errors are defined as follows:

$$\bar{e}_\beta = \frac{1}{t_{\text{sim}}} \int_0^{t_{\text{sim}}} |e_\beta(t)| dt \quad (34a)$$

$$\bar{e}_r = \frac{1}{t_{\text{sim}}} \int_0^{t_{\text{sim}}} |e_r(t)| dt, \quad (34b)$$

with

$$e_\beta(t) = \beta^* - \beta(t) \quad (35a)$$

$$e_r(t) = r^*(t) - r(t). \quad (35b)$$

where

$$\beta^* = 0 \quad (36a)$$

$$r^*(t) = \frac{v_x}{l} \delta. \quad (36b)$$

Note that the ideal yaw rate in this paper, $r^*(t)$, is chosen as the yaw rate value of a neutral steer vehicle.

As shown in Table 2, the average errors of the proposed method are remarkably lower than the errors of the other two. This quantitative comparison testifies the graphical results shown above. The results for $\delta = 0.12$ rad are not presented since the errors of the Ackerman method diverge.

4.2 Simulations with sinusoidal inputs

In this section, sinusoidal signals are utilised as steering inputs to the system. Here we first present the vehicle side-slip responses to a very intense sinusoidal steering input $\delta = 0.1 \sin \frac{\pi}{3} t$ rad in Figure 14. With a steer ratio of, say 1:12, the steering column is turned from -69° to $+69^\circ$ then back to -69° every 6 seconds at a vehicle speed of $v_x = 15$ m/s, which represents a highly challenging steering scenario. Figure 14 demonstrates that the vehicle side-slip curve produced by the proposed method is consistently lower (absolute

value) than the other two. Besides, from the yaw rate responses shown in Figure 15, we see that the yaw rate produced by the proposed method is very close to the ideal curve while the other two curves are not. Figure 16 shows that consistent with the step steering input situation, the slip ratio of the proposed method is larger in absolute value but always within a very small and safe range.

Similarly, when the sinusoidal steering input is pushed up to an even more challenging case $\delta = 0.13 \sin \frac{\pi}{3}t$ rad, the Ackerman method makes the vehicle states diverge and the vehicle loses its stability, as is seen from Figures 17–19. But with the proposed control scheme on-board, the electric vehicle is completely stable, and the vehicle side-slip response and yaw rate response still present themselves as the best among the three. The equal torque method, as expected, presents an intermediate performance.

Again in Table 2, we present the average errors of the vehicle side-slip angle and yaw rate for $\delta = 0.1 \sin \frac{\pi}{3}t$ rad. In this scenario, the average errors of the proposed method are also greatly lower than the errors of the other two. The results for $\delta = 0.13 \sin \frac{\pi}{3}t$ rad are not presented since the errors of the Ackerman method diverge.

In summary, the simulation results demonstrate that in response to challenging steering inputs (in the form of large steps and fast sinusoids), the proposed method outperforms the competing methods in terms of the vehicle side-slip angle and yaw rate responses. More precisely, the proposed method maintains vehicle side-slip angle very close to zero in both steering scenarios. Thus, the vehicle heading direction is kept very close to the vehicle velocity direction of the mass centre. Besides, yaw rate is pushed closer to the ideal value that corresponds to neutral steer, which attenuates the normal sluggish understeer characteristic. The improvements in vehicle side-slip angle and yaw rate increase the wheel slip (absolute value) of the inner-driving wheel, but the slip ratio still remains within a small and safe range.

5 Conclusions

In this paper, we demonstrated the mathematical relationships between the steady-state vehicle states (vehicle side-slip angle and yaw rate) and the motor torque difference. Based on these mathematical derivations, we designed an electronic differential control scheme that minimises vehicle side-slip angle as well as optimises yaw rate. Simulation results demonstrated that in challenging steering scenarios, the proposed method outperforms the commonly used equal torque method and Ackerman method, in terms of the vehicle side-slip angle and yaw rate responses. The handling of the electric vehicle is effectively enhanced through the reduction of vehicle side-slip angle and the optimisation of yaw rate. Meanwhile, the low slip ratio magnitudes neither jeopardise vehicle safety nor cause any excessive tyre wear.

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Tables

Table 1 Vehicle parameters of the electric racing car.

Parameter	symbol	Value
Vehicle Mass	m	318 kg
Rear Track Width	d_r	1.153 m
Distance	l_f	0.785 m
Distance	l_r	0.765 m
Roll Moment of Inertia	I_x	18.43 kg·m ²
Yaw Moment of Inertia	I_z	250 kg·m ²
Total Roll Stiffness	k	51500 Nm/rad
Total Roll Damping	c	6669 N·m·s
Centre of Gravity Height	h	0.28 m
Tire Radius	R	0.218 m
Motor Peak Power (each)	$P_{\max}$	30 kw

Table 2 Average errors of the vehicle side-slip and the yaw rate.

	$\delta = 0.1 \text{ rad}$		$\delta = 0.1 \sin \frac{\pi}{3} t \text{ rad}$	
	$\bar{e}_\beta \text{ (rad)}$	$\bar{e}_r \text{ (rad/s)}$	$\bar{e}_\beta \text{ (rad)}$	$\bar{e}_r \text{ (rad/s)}$
Equal Torque Method	0.01255	0.09607	0.01231	0.07812
Ackerman Method	0.01078	0.06999	0.01153	0.06089
Proposed Method	0.00392	0.00190	0.00662	0.03208

Figures

Figure 1 Top view of the vehicle local coordinate frame.

Figure 2 Ackerman steering geometry.

Figure 3 Force system acting on the vehicle.

Figure 4 Schematic of the proposed control system.

Figure 5 Third generation all-electric race car developed at our institution.

Figure 6 Vehicle side-slip responses to step input $\delta = 0.1$ rad.

Figure 7 Yaw rate responses to step input $\delta = 0.1$ rad.

Figure 8 Slip ratio responses of the inner-driving wheel to step input $\delta = 0.1$ rad.

Figure 9 Longitudinal tyre force responses of the inner-driving wheel to step input $\delta = 0.1$ rad.

Figure 10 Wheel angular velocity responses of the two driving wheels to step input $\delta = 0.1$ rad using the proposed method.

Figure 11 Vehicle side-slip responses to step input $\delta = 0.12$ rad.

Figure 12 Yaw rate responses to step input $\delta = 0.12$ rad.

Figure 13 Slip ratio responses of the inner-driving wheel to step input $\delta = 0.12$ rad.

Figure 14 Vehicle side-slip responses to sinusoidal input $\delta = 0.1 \sin \frac{\pi}{3} t$ rad.

Figure 15 Yaw rate responses to sinusoidal input $\delta = 0.1 \sin \frac{\pi}{3} t$ rad.

Figure 16 Slip ratio responses of the inner-driving wheel to sinusoidal input $\delta = 0.1 \sin \frac{\pi}{3} t$ rad.

Figure 17 Vehicle side-slip responses to sinusoidal input $\delta = 0.13 \sin \frac{\pi}{3} t$ rad.

Figure 18 Yaw rate responses to sinusoidal input $\delta = 0.13 \sin \frac{\pi}{3} t$ rad.

Figure 19 Slip ratio responses of the inner-driving wheel to sinusoidal input $\delta = 0.13 \sin \frac{\pi}{3} t$ rad.