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Electronic Differential Design for Vehicle Side-Slip Control

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Abstract—This paper introduces a novel electronic differential control method designed to adjust the vehicle side-slip angle. With the new electronic differential on board, the electric car with independent driving motors can achieve a next-to-zero side-slip angle, which is of great significance in enhancing vehicle handling. The proposed electronic differential is implemented in the form of a closed-loop control system that constantly regulates the torque commands sent to the independent driving motors. These commands are generated to tune the difference between the road-tire reaction forces at the amount associated with zero side-slip angle. Comparative simulations manifest that the proposed method outperforms the common equal torque scheme in various challenging steering scenarios.

I. INTRODUCTION

Many existing vehicle dynamic control solutions are designed for passenger cars with the focus to maintain the vehicle stability. More precisely, most control schemes are developed to keep stability-related quantities, for example wheel slip, in a “safe region” [1]–[3]. This is especially the focus of signal processing and control methods integrated to drive-by-wire systems [4]–[6]. However, some vehicle states such as the vehicle side-slip angle and the yaw rate, which are directly related to the dynamic performance of the vehicle, have less been the focus of vehicle stability methods. In this paper, we focus on designing a electronic differential controller that minimizes the vehicle side-slip. Such a controller is expected to always maintain the vehicle heading direction consistent with its velocity direction at the centre of gravity. The consistency of heading and velocity directions would provide the driver with an appropriate level of control during cornering.

We demonstrate a direct relationship between vehicle side-slip and the difference in the longitudinal road-tire reaction forces on the driven wheels. We then mathematically derive the desired difference in the road-tire reaction forces that would achieve zero vehicle side-slip and show that it is indirectly related to the difference in the driving motor torques. A closed-loop control system is then presented for controlling the motor torques to achieve zero side-slip.

In a number of simulations, we compare the side-slip angle of a typical electric car equipped with our proposed electronic differential, with the side-slip of the same car using the equal torque method [7]. The equal torque method is a common electronic differential control scheme based on which identical torque commands are sent to both driving motors. Our simulation results demonstrate that the proposed method endows the electric car with a close-to-zero side-slip angle (while maintaining its stability in terms of wheel slip ratio) in challenging steering scenarios whereas the competing scheme fails.

The outline of this paper is as follows. In section II, we present our proposed method. Comparative simulations are presented in section III, followed by conclusions in section IV.

II. SIDE-SLIP CONTROL VIA ELECTRONIC DIFFERENTIAL

Consider a rear-driven electric vehicle with a local coordinate frame attached to its centre of mass, as schematically shown in Fig. 1. The x-axis is the centreline of the vehicle and points forwards, while the y-axis points to the left. The x-y plane is parallel to the ground, and the z-axis is perpendicular to the x-y plane and points upwards. The origin of the coordinate system is normally at the vehicle centre of mass (C).

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Fig. 1. Top view of the vehicle local coordinate frame.
The following equations:

\[
\begin{align*}
\sum F_x &= m\ddot{x} - mv_y \gamma \\
\sum F_y &= m\ddot{y} + m\gamma v_x \\
\sum M_z &= I_{\gamma} \gamma \\
\sum M_x &= I_{\phi} \phi,
\end{align*}
\]

where,

\[m = \text{vehicle mass}, \quad v_x = \text{longitudinal velocity at the centre of mass}, \quad v_y = \text{lateral velocity at the centre of mass}, \quad \gamma = \text{yaw rate}, \quad p = \text{roll rate}, \quad I_z = \text{yaw moment of inertia}, \quad I_x = \text{roll moment of inertia}.\]

It is important to note that we use a vehicle roll dynamic model (instead of a planar one) to take into account the vehicle roll motion. Expanding the left-hand side of the last three equations in (1), we get the complete version of the equations of motion that govern the lateral, yaw and roll motion of the car:

\[
\begin{align*}
C_{\gamma} \gamma + C_p \phi + C_{\beta} \beta + C_{v} \varphi + C_{\delta} \delta &= m\ddot{y} + m\gamma v_x \\
E_{\gamma} \gamma + E_{p} \phi + E_{\beta} \beta + E_{v} \varphi + E_{\delta} \delta &= I_{\gamma} \gamma \\
D_{\gamma} \gamma + D_{p} \phi + D_{\beta} \beta + D_{v} \varphi + D_{\delta} \delta &= I_{\phi} \phi,
\end{align*}
\]

where,

\[\beta = \text{vehicle side-slip angle}, \quad \varphi = \text{roll angle}, \quad \delta = \text{cot-average steering angle}.\]

The naming of the angle \(\delta\) is due to the cotangent equation \(\cot \delta = (\cot \delta_1 + \cot \delta_2) / 2\). The coefficients that appear on the left-hand side of the equations are vehicle parameters that can be measured and are normally assumed to be time-invariant in vehicle dynamic analysis. These coefficients are explicitly expressed in [8].

When we have electronic differential on-board, we are able to send different torque commands to the two driving motors, so the reaction forces \(F_{r3}\) and \(F_{r4}\) can be different. Denoting the difference by \(\Delta F = F_{r3} - F_{r4}\), we note that such a difference is equivalent to an additional moment \(\Delta M = \Delta F \times d/2\) applied on the rear axle plus a force \(\Delta F\) exerted at the centre of the rear axle, as indicated in Fig. 2. Thus, in presence of a difference between the reaction forces, only the last equation in (2) needs to be modified as below:

\[
D_{\gamma} \gamma + D_{p} \phi + D_{\beta} \beta + D_{v} \varphi + D_{\delta} \delta + \frac{d}{2} \cdot \Delta F = I_{\phi} \phi. \quad (3)
\]

In the steady state, the equations of motion (2) are simplified to:

\[
\begin{bmatrix}
C_{\beta} & C_{\gamma} - mv_x \\
E_{\beta} & E_{\gamma} \\
D_{\beta} & D_{\gamma} \\
\end{bmatrix}
\begin{bmatrix}
\beta \\
\gamma \\
\varphi
\end{bmatrix}
= \begin{bmatrix}
-C_{\delta} & 0 \\
-E_{\delta} & 0 \\
-D_{\delta} & -\frac{d}{2}
\end{bmatrix}
\begin{bmatrix}
\delta \\
\Delta F
\end{bmatrix}.
\]

Solving the above system of equations, the following side-slip (steady state) response is derived in terms of the control inputs \(\delta\) and \(\Delta F\):

\[
\beta = \frac{Z_0}{Z_0} \delta + \frac{Z_F}{Z_0} \Delta F, \quad (5)
\]

where,

\[
Z_F = \frac{4}{3}(E_{\gamma}C_{\beta} - E_{\gamma}C_{\varphi} - mv_x E_{\varphi}) + E_{\varphi}(D_{\gamma}C_{\beta} - D_{\gamma}C_{\varphi} + mv_x D_{\varphi}) + E_{\varphi}(D_{\delta}C_{\beta} - D_{\delta}C_{\varphi} + mv_x D_{\varphi}) + E_{\delta}(D_{\gamma}C_{\beta} - D_{\gamma}C_{\varphi} + mv_x D_{\varphi}) + E_{\delta}(D_{\delta}C_{\beta} - D_{\delta}C_{\varphi} + mv_x D_{\varphi}).
\]

Equation (5) shows that the vehicle side-slip is directly related to the difference in the two reaction forces on the rear wheels, \(\Delta F\). By controlling the driving torques generated by the two motors, we can tune \(F_{r3}\) and \(F_{r4}\) to attain the desired difference, in such a way that the desired side-slip is achieved.

In steady-state cornering, without electronic differential on-board, the side-slip angle is normally non-zero. Naturally, drivers always assume that the vehicle heading direction is the direction where the vehicle is going, and this wrong assumption can mislead the driver into performing excessive or insufficient steering actions. Our approach is based on regulating the side-slip angle expressed by equation (5) at zero. As a result, the vehicle heading direction becomes consistent with the vehicle velocity direction, and this consistency improves vehicle handling and the driver’s control.

Substituting the desired vehicle side-slip \(\beta = 0\) in equation (5) leads to the following desired reaction force difference that would achieve zero side-slip:

\[
\Delta F^* = -\frac{Z_0}{Z_F} \delta. \quad (7)
\]

Equation (7) is the control goal which can be indirectly achieved by tuning the motor torques.

The road-tire reaction forces applied on the tire contact patches are related to the torques produced by the driving motors according to the following torque equilibrium equation in the driven wheel coordinate:

\[
T = J_{a} \frac{d \omega}{dt} + F_x r + F_z a, \quad (8)
\]

where,

\[T = \text{driving torque}, \quad J = \text{moment of inertia of the driven wheel}, \quad \omega = \text{wheel angular speed}, \quad r = \text{wheel radius}, \quad F_x = \text{road-tire reaction force exerted on the contact patch}, \quad F_z = \text{normal force applied by the ground}, \quad a = \text{pneumatic trail of the wheel}.
\]

Thus, the left and right reaction forces and their difference can be indirectly controlled by tuning the difference between actuator commands sent to the driving motors.

Fig. 3 shows a block diagram of the structure of the proposed electronic differential. The electronic differential
(E-diff) controller consists of two PID controller units. The “Side-slip Controller” generates half the difference in torque commands, \( \Delta T/2 \), from the side-slip error (the difference between the desired side-slip and its actual value), and the “Speed Controller” provides the base torque \( T_{\text{base}} \) which is the average of the two torque commands sent to the left and right motor.

We tune the base torque in such a way that the longitudinal speed of the vehicle, \( v_x \), follows the desired speed read from the throttle pedal sensor. The outputs of the two controllers are summed up and subtracted to form the torque commands to be sent to the left and right inverter. The two inverters convert torque commands to electric signals, in conjunction with the feedback phase signal \( \phi \) read from the motor encoders, to drive these two brushless permanent-magnet DC motors.

Several sensors are installed on the car to measure two vehicle states \( a_x, a_y \) and the driver’s throttle command \( v^*_x \). The vehicle side-slip angle is estimated from the lateral and longitudinal vehicle speed based on the definition of vehicle side-slip angle, \( i.e. \)

\[
\beta = \arctan\left(\frac{v_y}{v_x}\right) \quad (9)
\]

and the velocities are measured via integrating the acceleration measurements. The measured side-slip angle and longitudinal speed are fed back to form the errors for the two PID controllers.

It is important to note that in practice, integration of the noisy acceleration measurements (provided by the ac-
celerometers) would result in continuously growing errors in \(v_x\) and \(v_y\) measurements. Therefore, the estimated value of the longitudinal speed \(v_x\) (fed back to the speed controller) and the vehicle side-slip angle \(\beta\) (fed back to the side-slip controller) become increasingly erroneous and may cause the two controllers to fail. To avoid this, measurement errors in \(v_x\) and \(v_y\) need to be frequently reset. A practical mechanism for resetting the \(v_y\) measurement error is to continuously monitor the steering angle (returned by the steering angle sensor, not shown in Fig. 3) and as soon as it gets very small (close to zero), reset \(v_y\) to zero. We trust that for most of the times the vehicle is going straight, which means that \(v_y\) is almost zero. Only in steering scenarios does our proposed controller need to get activated, and as soon as the vehicle comes back to straight manoeuvring the \(v_y\) measurement will be reset. As for \(v_x\), we continuously monitor the throttle pedal position (returned by the throttle pedal sensor) and the average linear speed of the two driven wheels (returned by the wheel angular speed sensors, not shown in Fig. 3), and reset \(v_x\) to the average linear speed when the throttle is released and the average linear speed is low. This is because that, when the average linear speed is low, as soon as the throttle is released, the two driven wheels will be free rolling without large slips and the average linear speed will become an accurate estimation for \(v_x\). Thus, the measurement errors are continuously reset to zero and will not increase with time.

III. SIMULATION RESULTS

In order to verify the effectiveness of the proposed control scheme, we have conducted a number of simulations in MATLAB Simulink environment. We compared our method with the equal torque method in which both driving motors are sent identical control commands. Our simulations comprise two sections: simulation with step steering input and simulation with sinusoidal steering input. In each section, we have examined the side-slip performance of a fully simulated vehicle as well as the slip ratio of the inner-driven wheel which has the worst slip among four wheels.

A. Simulation with Step Input

In this section, step input is employed as steering input to the simulated vehicle. We have examined a large range of possible step magnitudes. For the sake of brevity, here we only present the simulation results for the step input \(\delta = 0.1\) rad. The initial vehicle longitudinal speed \(v_x\) is fixed at 15 m/s (54 km/h). At this speed, this is a challenging steering scenario which can mean the steering column is instantly turned \(60^\circ\) – assuming a 1:12 steer ratio. To clearly show the transients, in our simulations, the step steering command occurs at \(t = 10\) s.

Fig. 4 shows the vehicle side-slip angle versus time with our method compared with the equal torque method. We observe that both side-slip angle signals converge to a certain value very quickly after the steering input occurs, but the one representing our control method is far smaller than the one representing the equal torque method. Indeed, using our method, we gain a steady-state side-slip angle of about 0.003 rad, while with equal torque method, the side slip reaches almost five times as large. This result demonstrates that with our control method, the vehicle heading direction is significantly closer to the vehicle velocity direction at the centre of gravity, and the driver can handle the vehicle easier with a more accurate sense of steering.

Fig. 5 shows the slip ratio response of the inner-driven wheel using our method and the equal torque method respectively. The inner-driven wheel normally presents the worst wheel slip because it is considerably unloaded by the centrifugal force during cornering. The slip ratio when using the equal torque method is small and always positive, while the one using our method is negative and larger in absolute value. This result indicates that when employing the equal torque method the longitudinal road-tire reaction force applied on the inner-driven wheel is positive (forward) and very small, while with our method, a large (in absolute value) negative (backward) longitudinal road-tire reaction force is generated to decrease the side-slip angle. In fact, the about 10% (absolute value) slip ratio is normally the optimal slip ratio for most tires at which the best traction is provided and this slip ratio neither jeopardises vehicle safety nor causes any excessive tire wear.

B. Simulation with Sinusoidal Input

In this section, sinusoidal signal is utilized as steering input to the system. We have examined various combinations of magnitudes and frequencies for the steering commands. Here we present the side-slip response to the sinusoidal steering input \(\delta = 0.05\sin 0.5\pi t\) rad in Fig. 6. The comparative responses to other sinusoidal steering inputs are similar. With a steer ratio of 1:12, the steering column is turned from \(-35^\circ\) to \(+35^\circ\) then back to \(-35^\circ\) in every 4 seconds at \(v_x = 15\) m/s, and this represents a highly challenging steering scenario. Fig. 6 demonstrates that our method reduces the vehicle side-slip angle to almost half the value of the equal torque method.

In summary, the simulation results demonstrate that in response to challenging steering demands (in the form of large steps and fast sinusoids), our method outperforms the competing method in terms of the vehicle side-slip value. More precisely, our method maintains the vehicle side-slip angle very close to zero. Thus, the vehicle heading direction is kept very close to the vehicle velocity direction at the centre of gravity. This improvement in the side-slip angle increases the wheel slips but to a level at which suitable traction is provided without excessive tire wear.

IV. CONCLUSIONS

Besides the vehicle stability concern, the vehicle dynamic performance is also a very important issue to be considered in electronic differential design. In this paper, we introduced the development of a novel electronic differential design that minimizes the vehicle side-slip angle. This effect makes the vehicle heading direction consistent with its velocity direction at the centre of gravity, which provides the driver a realistic sense of handling. We mathematically demonstrated
how our new control method works to depress the vehicle side-slip. Simulation results showed that in challenging steering scenarios, our new method outperforms the common scheme of equal torque control.

Fig. 5. Slip ratio responses of the inner-driven wheel to the step input $\delta = 0.1$ rad.

Fig. 6. Vehicle side-slip responses to the sinusoidal input $\delta = 0.05 \sin 0.5\pi t$ rad.

REFERENCES


